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Spectral flow and boundary string field theory for angled D-branes

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Abstract: D-branes intersecting at an arbitrary fixed angle generically constitute a configuration unstable toward recombination. The reconnection of the branes nucleates at the intersection point and involves a generalization of the process of brane decay of interest to non-perturbative string dynamics as well as cosmology. After reviewing the string spectrum of systems of angled branes, we show that worldsheet twist superfields may be used in the context of Boundary Superstring Field Theory to describe the dynamics. Changing the angle between the branes is seen from the worldsheet as spectral flow with boundary insertions flowing from bosonic to fermionic operators. We calculate the complete tachyon potential and the low energy effective action as a function of angle and find an expression that interpolates between the brane-antibrane and the Dirac-Born-Infeld actions. The potential captures the mechanism of D-brane recombination and provides for interesting new physics for tachyon decay.

Keywords: D-branes, Tachyon Condensation, String Field Theory.

Contents

1. Introduction

Properties of non-BPS configurations of Dp-branes are very interesting for a number of reasons. For one, these are unstable dynamical systems and hence allow the study of time evolution for non-perturbative objects in string theory. With our now rather elaborate understanding of the spectrum and the interactions of the theory, a natural next step is then to study such dynamics. The presence of tachyonic modes in non-BPS systems allows a number of new scenarios: the tracking of tachyonic matter as it evolves in time, the formation of defects (lower dimensional BPS Dp-branes) as part of the mechanism of tachyon condensation, the testing of the intricate interconnections of closed string modes and open string dynamics. Besides the recent interest in tachyon rolling [1, 2] and S-branes [3, 4] in string theory, all these lend themselves to applications in cosmology as well.

In brane inflation, an inflationary scenario in the brane world describing the early universe, brane interactions and evolution provide a natural origin to the inflaton and its potential [5]–[12]. A particularly attractive scenario consists of a configuration of angled branes [7]. The inflationary epoch of such a system is relatively easy to study [8] and is captured by long distance physics. Toward the end of the inflationary epoch, when the separation between branes becomes small compared to the string scale, the lightest open string mode becomes tachyonic. As the branes start intersecting, a rolling tachyon describes the recombination of the branes, defect formation, and decay to closed and open string modes. This is a fundamental interaction process in string theory that is interesting in its own right: the evolutionary process by which intersecting branes reconnect to minimize free energy. Furthermore, the production of defects in brane inflation, which appear as cosmic strings, can be tested in the near future in cosmological observations [13]. This means that a stringy formulation of angled branes can be very useful for phenomenology and can serve as a guide to properly embed string theory into a cosmological scenario. In this paper, we present a step in this direction.

For parallel Dp-branes in type IIA(B) theories with p even (odd), one typically has a BPS stable system; the massless open string modes form a supersymmetric non-abelian gauge theory on the worldvolume, often written at low energies as a Dirac-Born-Infeld (DBI) action. On the other hand, for a non-BPS brane, or for a system of antiparallel Dp branes known as a brane-antibrane pair, one has a non-supersymmetric unstable configuration. The relevant effective actions [14]–[17] have been written in the framework of Boundary Superstring Field Theory (BSFT) [18]–[24]. Hence, to study the regime interpolating between a configuration of parallel and anti-parallel branes, it is natural to adopt BSFT techniques to write a low energy effective action describing branes at an arbitrary angle. As we change the angle between the two branes from 0 to π, we would flow from a DBI action for two parallel D branes (DD system) to the brane-antibrane action (D$\bar{\text{D}}$ system).

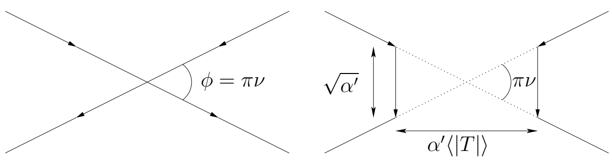

Figure 1: Angled branes and a crude description of their recombination.

For generic non-zero angles, space-time supersymmetry is completely broken. For small separation between the branes, the lightest open string mode is tachyonic and its mass runs as a function of angle. The gradual rotation of the branes is seen from the string worldsheet as spectral flow. As the angle increases from 0 to $\pi/2$, the worldsheet fermion in the plane of the angle flows from an NS to a Ramond fermion. Further increasing the angle to π makes the fermion flow back to be NS, but with the opposite GSO projection than at zero angle. Since the tachyonic mode is our main focus, we shall measure the angle ϕ, or $\nu \equiv \phi/\pi$, with respect to the brane-antibrane system; that is, $\nu = 0$ corresponds to the D$\bar{\text{D}}$ configuration, while $\nu = 1$ corresponds to the supersymmetric DD system.

For $0 \leq \nu \leq 1$, we are then dealing with worldsheet physics reminiscent of strings on orbifolds. The brane angle plays the role of the orbifold twist yielding worldsheet bosons and fermions with non-integer moding. In the case at hand, the twist ν can be an arbitrary irrational number. This introduces the language of twist fields in the BSFT technology requiring a careful treatment of the monodromy properties of worldsheet operators. Among other novelties, the presence of the twist fields implies that what used to be a free and explicitly calculable worldsheet theory becomes interacting, so only a perturbative expansion can be performed for some quantities. As a result of this complication, we can determine the effective action from BSFT with twists only up to the canonical kinetic term, but the tachyon potential can be obtained exactly.

Our main results can be summarized with the following expression for the low energy effective action of the angled D8-branes system.

$$
\begin{aligned}
S(T,\bar{T}) &= \int d^8x\ V_\nu(T)\sqrt{-\mathrm{P}[g]}\,\left(1+\mathcal{K}_\nu\ \partial_a T\partial^a\bar{T}\right)+\cdots \\
V_\nu(T) &= N_\nu \prod_{n=0}^{\infty} \frac{\sin\left(\dfrac{\sin(\frac{\pi\nu}{2})2\pi\alpha' T\bar{T}}{[n+\frac{1}{2}(\nu+1)]\,\mathrm{B}(n+1,\nu)}+\dfrac{\pi\nu}{2}\right)}{\sin\left(\dfrac{\sin(\frac{\pi\nu}{2})}{n\,\mathrm{B}(n,\nu)}2\pi\alpha' T\bar{T}+\dfrac{\pi\nu}{2}\right)}, \\
K_\nu &= 2\pi\alpha'^2\frac{\sqrt{\pi}\Gamma(1-\frac{\nu}{2})}{2^\nu\Gamma(\frac{3-\nu}{2})}\left\{1+(1-\nu)\left[\Psi(2-\nu)-\Psi\left(\frac{3-\nu}{2}\right)\right]\right\},
\end{aligned}
$$

in which $\mathrm{B}(p,q)\equiv\Gamma(p)\Gamma(q)/\Gamma(p+q)$ and $\Psi(y)\equiv\frac{d}{dy}\ln\Gamma(y)$ are the Beta and Polygamma functions respectively, $\mathrm{P}[g]$ is the pull-back of the metric to the brane world-volume, and 'a' runs over spacetime directions transverse to the plane of intersection. We argue on physical grounds that the potential is normalized to be

$$
N_\nu = 2\tau_8\sqrt{\frac{2\pi\alpha'}{\cos(\frac{\pi\nu}{2})}}\,,\qquad V_\nu(0)=2\tau_8\sqrt{\frac{2\pi\alpha'}{\cos(\frac{\pi\nu}{2})\sin(\frac{\pi\nu}{2})}}\,. \tag{1.1}
$$

Furthermore, some dependence on worldvolume gauge fields can be introduced in this action by covariantizing the spacetime derivatives.

These complicated expressions have the correct properties to interpolate between the unstable $\mathrm{D}\bar{\mathrm{D}}$ system and the BPS DD system. For small ν, the behaviour of the potential is well approximated by the first term in the infinite product. For $\nu<1$, the complex field T is tachyonic, and will roll to an expectation value of $\sqrt{2\pi\alpha'}\,\langle|T|\rangle\sim\sqrt{\cot(\frac{\pi\nu}{2})}$, at which point the potential achieves its minimal value of $V_\nu(\langle|T|\rangle)/V_\nu(0)\sim\sqrt{\sin(\frac{\pi\nu}{2})}$. Note that the minimum moves from $T\to\infty$ when $\nu=0$ to $T=0$ when $\nu=1$, agreeing with the expected physics of the system. Beyond the stable minimum of the potential, the potential increases and diverges at $2\alpha' T\bar{T}=(1-\nu/2)/\sin\pi\nu/2$. The kinetic term normalization ensures that the tachyon mass flows smoothly with ν from that of the BSFT $\mathrm{D}\bar{\mathrm{D}}$ tachyon to 0 for the stable DD system. Dynamics in this potential involves recombination of the branes (see figure 1), followed by the decay of the tachyon condensate - after the roll to the minimum of the potential where several channels are available for dumping energy into open string modes; for example, the gauge fields transverse to the angling plane incorporated in the low energy effective action by covariantizing the derivatives.

This paper is organized as follows: section 2 is a brief review of the spectrum of strings for the angled D-brane system. In section 3, we describe how the language of twist fields is a convenient tool to study angled branes, and we develop the boundary twist field formalism to apply to the BSFT of branes at angles. We find that the twist fields flow from being bosonic to fermionic boundary fields through worldsheet spectral flow. Section 4 is a review of the BSFT for the brane-antibrane system, and in section 5 we perform BSFT calculations for angled branes, and derive the tachyon low energy effective action. In section 6, we illustrate that our results give an effective description of the recombination process. We discuss the results and future work in section 7. Appendix is a summary of our superspace and CFT conventions.

2. Spectrum of angled branes

In this section, we review the spectrum of strings stretching between two D-branes of the same dimension intersecting at an arbitrary angle [25]–[27]. For simplicity we work with two D8-branes in IIA theory angled in the 8-9 plane. In this section, the worldsheet is represented by the strip parameterized by (τ,σ) with boundaries located at $\sigma = 0, \pi$. We introduce a complex structure in the 8-9 directions with new coordinates $Z = 2^{-1/2}[X^8(z,\bar{z}) + iX^9(z,\bar{z})]$, $\Psi(z) = 2^{-1/2}[\psi^8(z) + i\psi^9(z)]$, where the ψ's are worldsheet fermions in the NSR formalism. With one brane lying along the X^8 direction and the other tilted by an angle $\phi \equiv \nu\pi$, the boundary conditions on Z for a string stretching between the branes are

$$\text{at} \quad \sigma = 0 \quad \begin{cases} \partial_\sigma \operatorname{Re} Z = 0\,, \\ \operatorname{Im} Z = 0\,, \end{cases} \qquad \text{at} \quad \sigma = \pi \quad \begin{cases} \partial_\sigma \operatorname{Re}\left(e^{i\pi\nu} Z\right) = 0\,, \\ \operatorname{Im}\left(e^{i\pi\nu} Z\right) = 0\,, \end{cases}$$

and that for the worldsheet fermion ψ is

$$\Psi(\tau, 0) = e^{-2\pi i\nu'}\Psi(\tau, 2\pi)\,.$$

We have used the standard doubling trick to extend the fermions on $\sigma = [0,\pi]$ to $\sigma = [0, 2\pi)$. Note that $0 \leq \nu \leq 1$, with the configuration corresponding to $\nu = 0$ being conventionally the brane-antibrane system and $\nu = 1$ corresponding to the brane-brane system. The twist ν' is $\nu + 1/2$ for NS sector fermions and ν for Ramond sector fermions.

Transforming to the upper half plane (UHP) through $z = e^{\sigma + i\tau}$, with the boundary now being along the real axis, the mode expansions for the free string become

$$\begin{aligned} Z(z,\bar{z}) &= i\left(\frac{\alpha'}{2}\right)^{1/2} \sum_{n\in\mathbb{Z}} \left[\frac{\alpha^\dagger_{n-\nu}}{(n-\nu)z^{n-\nu}} + \frac{\alpha_{n+\nu}}{(n+\nu)\bar{z}^{n+\nu}}\right], &\qquad \Psi(z) &= \sum_{r\in\mathbb{Z}-\nu'} \frac{\psi_r}{z^{r+1/2}}\,, \\ Z^\dagger(z,\bar{z}) &= i\left(\frac{\alpha'}{2}\right)^{1/2} \sum_{n\in\mathbb{Z}} \left[\frac{\alpha_{n+\nu}}{(n+\nu)z^{n+\nu}} + \frac{\alpha^\dagger_{n-\nu}}{(n-\nu)\bar{z}^{n-\nu}}\right], &\qquad \Psi^\dagger(z) &= \sum_{s\in\mathbb{Z}+\nu'} \frac{\psi^\dagger_s}{z^{s+1/2}}\,. \end{aligned} \tag{2.1}$$

For these expansions, the negative real axis has one boundary condition, and the positive real axis the other; hence there is a flip in boundary conditions at $z = 0$ and $z = \infty$. Quantization of the string leads to the oscillator algebra

$$\left[\alpha_{m+\nu}, \alpha^\dagger_{n-\nu}\right] = (m+\nu)\delta_{n+m=0}\,, \qquad \left\{\psi_r, \psi^\dagger_s\right\} = \delta_{r+s=0}\,.$$

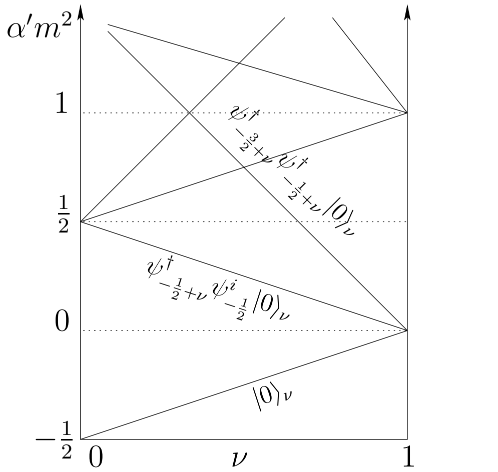

Figure 2: The NS+ spectrum of the states stretched between a brane pair at an angle $\phi = \pi\nu$. From the brane-antibrane system $\nu = 0$, the spectrum flows to the NS+ sector for the brane-brane system at $\nu = 1$. $\psi^i_{-1/2}$ are the fermionic raising operators in the transverse directions in which the branes are parallel.

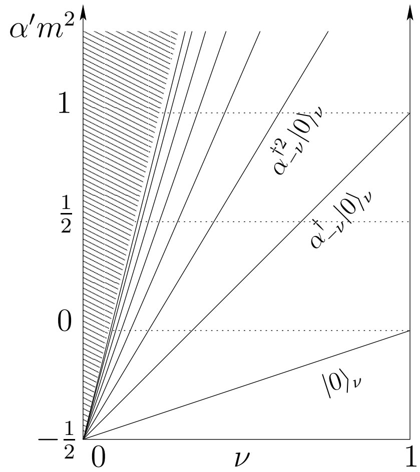

Figure 3: The bosonic excitations of the tachyonic NS vacuum which become momentum modes when $\nu = 0$; the states of different masses correspond to KK-like excitations of the lowest state, and the spectrum is indicative of an expansion in states localized about the intersection. $|0\rangle_\nu$ is the lowest lying tachyon that we focus on in this work.

To compute the spectrum, we note the relations

$$\begin{aligned} \left[\alpha^{\dagger}_{n-\nu}, L_0\right] &= (n-\nu)\alpha^{\dagger}_{n-\nu}\,, & \left[\alpha_{n+\nu}, L_0\right] &= (n+\nu)\alpha_{n+\nu}\,, \\ \left[\psi_r, L_0\right] &= r\psi_r, & \left[\psi^{\dagger}_s, L_0\right] &= s\psi^{\dagger}_s\,, \end{aligned} \tag{2.2}$$

with $L_0 = (\sum_{n\in\mathbb{Z}} : \alpha^{\dagger}_{n-\nu}\alpha_{-n+\nu} : + \sum_{r\in\mathbb{Z}-\nu'} r : \psi^{\dagger}_{-r}\psi_r :) + E_0$, where the total zero point energy is

$$E_0 = \frac{1}{2}(\nu - 1) \quad \text{in the NS sector;}\, E_0 = 0 \quad \text{in the R sector}\,.$$

We then define the twisted vacuum $|0\rangle_\nu$ by

$$\begin{aligned} \alpha^{\dagger}_{n-\nu}|0\rangle_\nu &= 0, \quad n \geq 1\,, & \psi_{n-\nu'}|0\rangle_\nu &= 0, \quad n \geq 1\,, \\ \alpha_{m+\nu}|0\rangle_\nu &= 0\,, \quad m \geq 0\,, & \psi^{\dagger}_{m+\nu'}|0\rangle_\nu &= 0\,, \quad m \geq 0\,. \end{aligned} \tag{2.3}$$

Note that flowing from $\nu = 0$ through $\nu = 1/2$, $|0\rangle_\nu$ becomes the first excited state in the NS sector, and $\psi_{-\frac{1}{2}+\nu}\,|0\rangle_\nu$ becomes the state of lowest energy.

We consider the spectrum starting with the brane-antibrane (NS−) spectrum at $\phi = \pi\nu = 0$ which flows to the brane brane (NS+) spectrum at $\phi = \pi\nu = \pi$. For $\nu > 0$ there is a splitting of the spectrum since the ψ oscillators in the different directions have different weights. The NS spectrum can be easily computed from the results above and is described in figure 2.

It is important to note also that the bosonic oscillators, $\alpha^{\dagger}_{-\nu}$ and $\alpha_{-1+\nu}$ are momentum oscillators at $\nu = 0$ and $\nu = 1$ respectively. Close to the brane-antibrane ($\nu = 0$) the modes created by $\alpha^{\dagger}_{-\nu}$ acting on the lowest tachyonic mode can still be tachyonic, as shown in

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figure 3. As the angle is decreased, additional tachyonic states appear at discrete intervals of the angle given by $\nu = 1/(2n+1)$ with $n = 0, 1, 2, \ldots$ For $\nu \neq 0$ or 1 these states are localized near the intersection point of the branes. Their masses decrease with ν as shown in figure 3 until they all become degenerate with the lowest lying tachyon at $\nu = 0$. Hence, we have an infinite tower of tachyonic states collapsing on top of each other at $\nu = 0$, reminiscent of decompactification in Kaluza Klein theories. In this case, we can think of the process as decompactification along the brane worldvolume: when the branes become antiparallel at $\nu = 0$, the tachyon is no more localized at an intersection point and can travel along the worldvolume. As we will see, the localization of this tower of tachyonic states helps in smoothing out the shape of the bent branes at their intersection point as they undergo recombination. Finally, note also that a mirror picture takes hold on the other side at $\nu = 1$. In that region, a infinite number of massive states collapse to zero mass comprising the modes of (gauge) fields localized at the intersection point.

3. Boundary twist fields

The operator formalism used to compute the spectrum in the previous section can be used to compute correlators in the angled brane system when one can arrange for the boundary condition flips to occur at $z = 0$ and $z = \infty$ along the boundary. In the BSFT formalism, it will be necessary to consider a condensate of perturbations located at arbitrary points. The operator formalism hence quickly becomes cumbersome and it is more convenient to adopt a different approach.

The problem of angled branes can be recast in the language of twist fields that incorporate the change in the moding of the worldsheet fields when acting upon twisted vacua. Such twist operators must be present when boundary conditions flip locally across insertion points to move from one endpoint of the open string located on one brane to the other [28]. We will first review the use of such operators as they have risen in the past in the context of studying strings on orbifolds. We will then move onto applying the technology to the system of angled branes and BSFT.

3.1 Review of twist fields

The expectation value of a stress energy tensor in a twisted vacuum is non-vanishing [29] and can be easily computed

$$\left\langle T_B^Z(z)\right\rangle = \frac{\frac{1}{2}\nu(1-\nu)}{z^2}\,{}_\nu\langle 0|0\rangle_\nu\ , \qquad \left\langle T_B^\Psi(z)\right\rangle = \frac{\frac{1}{2}(\nu'-\frac{1}{2})^2}{z^2}\,{}_\nu\langle 0|0\rangle_\nu\ .$$

The double poles at $z = 0$ suggest that there is a local source of stress-energy at $z = 0$ whose origin is a local operator. These new operators of conformal dimension $h = 1/2\nu$ in the NS sector (with $\nu' = 1/2 + \nu$) or $h = 1/8$ in the R sector (with $\nu' = \nu$) factor the information about the twist away from the vacuum. One then defines bosonic twist operators $\sigma^{\pm}$ having the OPEs (on the UHP) [29][1]

$$\begin{aligned}
\partial Z(z)\sigma^{+}(0) &= \frac{1}{z^{1-\nu}}\mu^{+}(0)\,, & \bar\partial Z(\bar z)\sigma^{+}(0) &= \frac{1}{\bar z^{\nu}}\,\mu'^{+}(0)\,,\\
\partial Z^{\dagger}(z)\sigma^{+}(0) &= \frac{1}{z^{\nu}}\,\mu'^{+}(0)\,, & \bar\partial Z^{\dagger}(\bar z)\sigma^{+}(0) &= \frac{1}{\bar z^{1-\nu}}\mu^{+}(0)\,,\\
\partial Z(z)\sigma^{-}(0) &= \frac{1}{z^{\nu}}\,\mu'^{-}(0)\,, & \bar\partial Z(\bar z)\sigma^{-}(0) &= \frac{1}{\bar z^{1-\nu}}\mu^{-}(0)\,,\\
\partial Z^{\dagger}(z)\sigma^{-}(0) &= \frac{1}{z^{1-\nu}}\mu^{-}(0)\,, & \bar\partial Z^{\dagger}(\bar z)\sigma^{-}(0) &= \frac{1}{\bar z^{\nu}}\,\mu'^{-}(0)\,.
\end{aligned} \tag{3.1}$$

These OPEs are determined as in [29] (where ν was a rational number) by the requirements that Z acquires a phase of $e^{\pm i\nu\pi}$ across the change in boundary condition implemented by $\sigma^{\pm}$; that the bosonic derivative fields create excited states; and that $\sigma^{\pm}$ are to be fields of highest weight. Since in this analysis all twist fields are to be placed on the worldsheet boundary, these relations hold for any $\nu \in [0,1]$, rational or otherwise, since we will never encounter ambiguous contour integrals involving branch cuts on the z-plane. Hence, the conformal weights of these twist and excited twist fields are

$$h_{\sigma^{\pm}} = \frac{\nu}{2}(1-\nu)\,, \qquad h_{\mu^{\pm}} = \frac{\nu}{2}(3-\nu)\,, \qquad h_{\mu'^{\pm}} = 1 - \frac{\nu}{2} - \frac{\nu^2}{2}\,.$$

Similarly, there are operators $s^{\pm}$ which twist the worldsheet fermions which have the OPEs

$$\begin{aligned}
\Psi(z)s^{+}(0) &= z^{\nu'-1/2}u'^{+}(0)\,, & \tilde\Psi(\bar z)s^{+}(0) &= \frac{1}{\bar z^{\nu'-1/2}}u^{+}(0)\,,\\
\Psi^{\dagger}(z)s^{+}(0) &= \frac{1}{z^{\nu'-1/2}}u^{+}(0)\,, & \tilde\Psi^{\dagger}(\bar z)s^{+}(0) &= \bar z^{\nu'-1/2}u'^{+}(0)\,,\\
\Psi(z)s^{-}(0) &= \frac{1}{z^{\nu'-1/2}}u^{-}(0)\,, & \tilde\Psi(\bar z)s^{-}(0) &= \bar z^{\nu'-1/2}u'^{-}(0)\,,\\
\Psi^{\dagger}(z)s^{-}(0) &= z^{\nu'-1/2}u'^{-}(0)\,, & \tilde\Psi^{\dagger}(\bar z)s^{-}(0) &= \frac{1}{\bar z^{\nu'-1/2}}u^{-}(0)\,.
\end{aligned} \tag{3.2}$$

These are again determined by requiring that Ψ picks up the appropriate phase, with the "+" and "−" fields twisting the fermions in opposite directions. The OPEs are given for both NS ($\nu' = \nu + 1/2$) and R ($\nu' = \nu$) fermions. Henceforth, we shall focus only on the NS sector and set $\nu' = \nu + 1/2$ as this entails the interesting dynamics of branes at angles. Note that by bosonizing the fermions, we may represent the fermionic sector in bosonic variables in the standard way

$$\begin{aligned}
&\Psi(z) \sim e^{iH(z)}\,, && \Psi^{\dagger}(z) \sim e^{-iH(z)}\,,\\
&s^{\pm}(z) \sim e^{\pm i\nu H(z)}\,, && u^{\pm}(z) \sim e^{\mp i(1-\nu)H(z)}\,, && u'^{\pm}(z) \sim e^{\pm i(1+\nu)H(z)}\,,
\end{aligned} \tag{3.3}$$

where H is a holomorphic bosonic field. The anti-holomorphic side has similar bosonization. The conformal weights of the twist fields s and u are then given by

$$h_{s^{\pm}} = \frac{\nu^2}{2}\,, \qquad h_{u^{\pm}} = \frac{1}{2}(1-\nu)^2\,, \qquad h_{u'^{\pm}} = \frac{1}{2}(1+\nu)^2\,.$$

[1]Because we are working with open strings, the holomorphic and anti-holomorphic excited twist fields are identified, whereas for the orbifold twist fields of [29], they are distinct.

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To construct the boundary condition changing operator, we require a field which twists both the bosons and the fermions. In [29], it is argued that in a supersymmetric theory, the twist on Ψ must compensate the twist on Z in order that the worldsheet supercurrent T_F is single or double valued in the NS and R sectors respectively. The operator of lowest dimension which accomplishes this is $\mathcal{T}_0^{\pm} \equiv (\sigma s)^{\pm}$ (although other combinations of twist fields which satisfy this condition are also important in this system; see section 3.4 for an example). These twist fields have a non-trivial supersymmetry transformation so they can be written as the lower component of a twist superfield. Since we work on a worldsheet with a boundary and the twist operators are to be placed only on the boundary, they must represent $\mathcal{N}=1$ supersymmetry rather than the $\mathcal{N}=(1,1)$ supersymmetry of the worldsheet bulk. Hence, $\mathcal{N}=1$ superfields will be used as BSFT boundary insertions that preserve the boundary supersymmetry while breaking conformal invariance.

Using the conventions of [27] (which differ from those in [29]), our twist superfield becomes

$$\boldsymbol{\mathcal{T}}^{\pm} \equiv \mathcal{T}_0^{\pm} + i\boldsymbol{\vartheta}\mathcal{T}_1^{\pm} \equiv (\sigma s)^{\pm} - \frac{i}{\sqrt{2\alpha'}}\boldsymbol{\vartheta}(\mu u)^{\pm}\,. \tag{3.4}$$

See appendix for details on deriving this expression. The top component of the twist superfield $\mathcal{T}_1$ has conformal weight $h = \frac{1}{2}(\nu+1)$. This is essentially the holomorphic side of the twist superfield constructed in [29], which in the present case is linked to the anti-holomorphic side through the boundary conditions.

3.2 Spectral flow

In inserting perturbations on the open string worldsheet boundary, it is convenient to represent the worldsheet by the unit disk. We will then consider the worldsheet as being defined by $|z|<1$ with boundary $|z|=1$. We parametrize this disk by $z=e^{-i\tau+\sigma}$ such that τ is an angular coordinate along the boundary. In the correspondences to follow we map the fields to the boundary of the unit disk with this coordinate τ, so that holomorphic and anti-holomorphic fermions are linked by the boundary conditions as described in appendix, and the derivatives are restricted to τ derivatives along the boundary.

It is evident from the OPEs (3.2), and particularly the bosonization (3.3), that in the limiting cases of no twist $\nu=0$ and a full twist $\nu=1$ the fermionic twist fields flow as

	$\nu=0$	$\longrightarrow$	$\nu=1$
s^+, s^- :	$\mathbb{1}$	$\longrightarrow$	$\Psi, \Psi^\dagger$
u^+, u^- :	$\Psi^\dagger, \Psi$	$\longrightarrow$	$\mathbb{1}$
u'^+, u'^- :	Ψ, $\Psi^\dagger$	$\longrightarrow$	$:\!\dot{\Psi}\Psi\!:$, $:\!\dot{\Psi}^\dagger\Psi^\dagger\!:$

Similarly, the limiting cases of the bosonic twist fields can be deduced from the OPEs; this is more subtle without the aid of a free field description for the twist fields, but the OPEs are sufficient to identify the endpoints of the flow. Although the resultant identifications are not unique the general structure can be obtained. When $\nu \to 0$, there remains a singularity in the OPE of ∂Z with σ^+. This identifies σ^+ with some combination of $:\!(Z^\dagger)^n\!:$ as the only structure to have a non-trivial simple pole in the OPE with ∂Z. Furthermore, since the operator multiplying that pole is just $n{:}(Z^\dagger)^{n-1}{:}$, μ^+ is also identified in the limit $\nu \to 0$. We then propose the mappings

$$\begin{array}{rccc}
 & \nu=0 & \longrightarrow & \nu=1 \\
\sigma^+ : & :(Z^\dagger)^n: & \longrightarrow & :(Z)^n: \\
\mu^+ : & -(2\alpha')n:(Z^\dagger)^{n-1}: & \longrightarrow & :\partial Z(Z)^n: \\
\mu'^+ : & :\partial Z^\dagger(Z^\dagger)^n: & \longrightarrow & -(2\alpha')n:(Z)^{n-1}:,
\end{array}$$

where $n \geq 0$. The relations satisfied by the "$-$" twist fields are obtained by swapping Z and $Z^\dagger$ in the table above. Note that for brevity we have now adopted a new notation for the ∂ symbol: $\partial Z \equiv 2^{-1/2}(\dot{X}^8 + i(X^9)')$, where the prime is the normal derivative to the boundary. The other derivatives are zero as dictated by the boundary conditions. The UHP boundary conditions on the fields are actually encoded within the OPEs (3.1), since their $\nu = 0$ (1) cases identify $\partial Z^\dagger(\partial Z)$ with $\bar{\partial} Z(\bar{\partial} Z^\dagger)$. The $n = 0$ case of the relations above is then

$$\begin{array}{rccc}
 & \nu=0 & \longrightarrow & \nu=1 \\
\sigma^\pm : & \mathbb{1} & \longrightarrow & \mathbb{1} \\
\mu^+,\mu^- : & 0 & \longrightarrow & \partial Z,\partial Z^\dagger \\
\mu'^+,\mu'^- : & \partial Z^\dagger,\partial Z & \longrightarrow & 0\,.
\end{array}$$

We argue that the $n = 0$ case is the correct limiting behaviour for the bosonic twist fields; if the twist fields $\sigma^\pm$ are to represent the boundary condition changing operators, then in the no-twist and full-twist cases they must reduce to the identity operator, since the boundary conditions on both branes are identical. In the next section we construct vertex operators under this assumption, and see that they exactly reproduce the tachyon and gauge field vertex operators in the corresponding limits. The formalism to follow is consistent irrespective of whichever n is chosen, and the flow of the fermions established from the bosonization guides the structure.

Combining the fermion and the $n = 0$ boson limiting expressions, we obtain the limiting expressions on the twist superfields:

$$\begin{array}{rccc}
 & \nu=0 & \longrightarrow & \nu=1 \\
\boldsymbol{\mathcal{T}}^+ = \mathcal{T}_0^+ + i\boldsymbol{\vartheta}\mathcal{T}_1^+ : & \mathbb{1} + \boldsymbol{\vartheta}0 & \longrightarrow & -\dfrac{i}{\sqrt{2\alpha'}}(i\sqrt{2\alpha'}\Psi + \boldsymbol{\vartheta}\partial Z) \\
\boldsymbol{\mathcal{T}}^- = \mathcal{T}_0^- + i\boldsymbol{\vartheta}\mathcal{T}_1^- : & \mathbb{1} + \boldsymbol{\vartheta}0 & \longrightarrow & -\dfrac{i}{\sqrt{2\alpha'}}(i\sqrt{2\alpha'}\Psi^\dagger + \boldsymbol{\vartheta}\partial Z^\dagger).
\end{array}$$

These assignments agree with all the OPEs, and they will be shown to enable the BSFT reconstruction of the flow of the states in the spectrum from GSO odd to GSO even states described in figures 2 and 3.

As noted earlier, the GSO$-$ vacuum for $\nu < 1/2$ becomes the GSO+ first excited state for $\nu > 1/2$, which is generated by a fermionic excitation of the ground state. This is the spectral flow described in [30], and shall be manifest on the worldsheet through the flow of the twist fields from bosonic to fermionic statistics. Thus the tachyon will become part of the U(2) gauge field which is the string state excited by a fermionic oscillator from the vacuum, in the off-diagonal Chan-Paton sector.

3.3 Twist field correlators

In the BSFT of angled branes, since one inserts the twist operators we encountered in the previous section on the boundary of the worldsheet at $|z| = 1$, correlators of the twist operators in the free conformal theory are needed. We can write the correlator

$$\left\langle \mathcal{T}_0^-(z_1)\mathcal{T}_0^+(z_2)\right\rangle = \frac{1}{(z_1 - z_2)^\nu} . \tag{3.5}$$

The power of $(z_1 - z_2)$ is fixed by the conformal weight of the $\mathcal{T}_0^\pm$ operators (each being $1/2\nu$), and the coefficient is fixed by the fact that in the $\nu \to 0$ and 1 limits, the trivial correlator of $\mathbb{1}$ and the fermion propagator are to be reproduced. In principle we can multiply this correlator by any non-singular function of ν which is 1 when $\nu = 0$ and 1, but any such additional ν dependence can be removed by rescaling the twist fields. As written (3.5) flows smoothly between being trivial for $\nu = 0$, to the correlator of a free fermion at $\nu = 1$.

Changing coordinates to τ, the angular coordinate along the disk boundary, we then have

$$\left\langle \mathcal{T}_0^-(\tau_1)\mathcal{T}_0^+(\tau_2)\right\rangle = \frac{(-iz_1)^{\frac{1}{2}\nu}(-iz_2)^{\frac{1}{2}\nu}}{(z_1 - z_2)^\nu} \tag{3.6}$$

$$= \frac{1}{\left[2\sin\left(\frac{\tau_1-\tau_2}{2}\right)\right]^\nu} . \tag{3.7}$$

The correlator we computed naturally incorporates time ordering as necessitated by a path integral formulation. When decomposing this correlator in frequency modes, unlike correlators of half integer dimension operators there are two branch choices and we need to be careful to pick branches consistent with the time ordering of $\mathcal{T}_0^-$ and $\mathcal{T}_0^+$. With $\epsilon \in (0, 2\pi)$, we identify two possible cases:

$$\begin{aligned}\left\langle \mathcal{T}_0^-(\epsilon)\mathcal{T}_0^+(0)\right\rangle &= e^{-i\frac{\pi}{2}\nu}\sum_{n=0}^{\infty}\frac{e^{i\epsilon(n+\frac{1}{2}\nu)}}{n\,\mathrm{B}(n,\nu)} ,\\ \left\langle \mathcal{T}_0^-(2\pi-\epsilon)\mathcal{T}_0^+(0)\right\rangle &= \left\langle \mathcal{T}_0^+(0)\mathcal{T}_0^-(-\epsilon)\right\rangle = \left\langle \mathcal{T}_0^+(\epsilon)\mathcal{T}_0^-(0)\right\rangle = e^{i\frac{\pi}{2}\nu}\sum_{n=0}^{\infty}\frac{e^{-i\epsilon(n+\frac{1}{2}\nu)}}{n\,\mathrm{B}(n,\nu)} ,\end{aligned} \tag{3.8}$$

where $\mathrm{B}(p,q)$ is the Beta function. We have associated positive frequency modes with $\mathcal{T}_0^-$ being ahead of $\mathcal{T}_0^+$, and negative frequency modes for the other ordering. In this context, it is being assumed that the expansions are made convergent by adding a small imaginary part to ϵ as appropriate for each of the two branches. This representation will be useful later when we will need the correlators in the interacting theory. We then have[2]

$$\left\langle \mathcal{T}_0^\mp(-\epsilon)\mathcal{T}_0^\pm(0)\right\rangle = e^{\mp i\pi\nu}\left\langle \mathcal{T}_0^\mp(\epsilon)\mathcal{T}_0^\pm(0)\right\rangle .$$

[2]It is worthwhile noting that this issue is slightly more subtle in the case at hand, as opposed to the special situation when $\nu = 1$ addressed in the literature. The subtlety arises as frequency modes $n + (\nu/2)$ with $n \in \{-\infty, \infty\}$ are different from $-(n + (\nu/2))$ when ν is not zero or one.

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The phase can be understood in terms of the phase some quantities acquire under a change in boundary conditions.

Calculating the correlator on the disk boundary of the excited twist operators in the same way, we get

$$\begin{aligned}\left\langle \mathcal{T}_1^-(z_1)\mathcal{T}_1^+(z_2)\right\rangle &= \frac{\nu}{(z_1-z_2)^{\nu+1}},\\ \left\langle \mathcal{T}_1^-(\tau_1)\mathcal{T}_1^+(\tau_2)\right\rangle &= \nu\frac{(-iz_1)^{\frac{1}{2}(\nu+1)}(-iz_2)^{\frac{1}{2}(\nu+1)}}{(z_1-z_2)^{\nu+1}}\\ &= \frac{\nu}{\left[2\sin\left(\frac{\tau_1-\tau_2}{2}\right)\right]^{\nu+1}}.\end{aligned} \tag{3.9}$$

As $\nu : 0 \to 1$ this correlator smoothly flows from zero to $\frac{1}{2\alpha'}\left\langle \partial Z \partial Z^\dagger\right\rangle$. The normalization of this correlator was also calculated in [31] by different methods.

3.4 Twisted vertex operators

By considering scattering amplitudes for angled branes,[3] it is easily seen that the strings stretching between the branes carry the boundary condition changing twist operators, whereas all other types of strings in the system do not. The vertex operators for the strings straddling the branes represent the tachyon. For scattering amplitude calculations, whether the unexcited or excited twist fields are utilized depends on the picture adopted. However for BSFT calculations, all operators must be expressed as worldsheet superfields, so it is most convenient to write the relevant vertex operators as such.

Given that $\boldsymbol{\mathcal{T}}$ has conformal dimension $\frac{1}{2}\nu$, an appropriate guess for the vertex operator corresponding to the lowest tachyonic state $|0\rangle_\nu$ of figure 2 is

$$\mathcal{V}_{\mathcal{T}+} = T\boldsymbol{\mathcal{T}}^+ e^{ik\cdot\boldsymbol{X}} = T\left[\mathcal{T}_0^+ + i\boldsymbol{\vartheta}\left(\mathcal{T}_1^+ - \sqrt{2\alpha'}\mathcal{T}_0^+ k\cdot\psi\right)\right]e^{ik\cdot X}.$$

X and ψ are the worldsheet fields in the directions transverse to the twisting plane. This operator has the correct properties to be identified with the (lowest) tachyon:

- The BRST quantization condition that a vertex operator must have total conformal dimension 1/2 (before ghost contributions) gives the state the correct mass: $\alpha' m_{\mathcal{T}}^2 = \frac{1}{2}(\nu-1)$.
- When $\nu = 0$ this becomes the well-known vertex operator for the NS$-$ vacuum, the D$\bar{\text{D}}$ tachyon.
- When $\nu = 1$, since $\boldsymbol{\mathcal{T}} \to \mathbf{D}_{\boldsymbol{\vartheta}}\boldsymbol{Z}$, the vertex operator becomes that of the gauge field or more correctly, a combination of the off-diagonal parts of the gauge field along one of the brane directions, A_8, and the transverse scalar, Φ. Such a linear combination of these fields is that state which is expected to become tachyonic from low energy analysis [34], with other linear combinations being massive. The coefficient T would then be relabeled as $\frac{1}{\sqrt{2}}(A_8 - i\Phi)$.

[3][32, 33] performed related calculations in Dp-Dq systems.

Vertex operators corresponding to other states in figures 2 and 3 can be constructed similarly; for instance states in the NS$-$ spectrum at $\nu = 0$ which include a $\mathbf{D}_{\boldsymbol{\vartheta}} \boldsymbol{Z}^{(\dagger)}$ excitation (such as the second lowest state in figure 2) include factors of the $h = \frac{1}{2} - \frac{1}{2}\nu$ superfield

$$(u\sigma)^{\pm} - \frac{i}{\sqrt{2\alpha'}} \boldsymbol{\vartheta} (s\mu')^{\pm}$$

which flows from $\mathbf{D}_{\boldsymbol{\vartheta}} \boldsymbol{Z}^{(\dagger)}$ to $\mathbb{1}$. Again the spectral flow is manifest in the change of statistics of the operator: from fermionic in the NS$-$ sector to bosonic in the NS+ sector.

4. BSFT of the brane-antibrane system

Before applying our formalism to a system of branes at an angle, it is useful to summarize the special case of brane-antibrane as described in the context of BSFT and presented in [15, 16]. We follow closely the conventions of [16] and [27]. We first restrict attention to D9-branes in type IIB theory. BSFT allows one to extract from the worldsheet sigma-model [35] - deformed at the boundary with relevant (*i.e.* off-shell) perturbations - the low energy effective action of spacetime fields [18, 19, 22, 23]. This framework developed for the bosonic sigma-model was extended to the superstrings in [14] and formally justified in [24, 36].

In the NS sector the spacetime action is given in the BSFT formalism by

$$S_{\text{spacetime}} = -\int \mathcal{D}X \mathcal{D}\psi \mathcal{D}\tilde{\psi}\; e^{-S_\Sigma - S_{\partial\Sigma}}. \tag{4.1}$$

where Σ is the worldsheet disk and $\partial\Sigma$ is its boundary. The worldsheet action in the bulk is typically

$$S_\Sigma = \frac{1}{4\pi}\int d^2z \,\left[\left(\frac{2}{\alpha'}\right)\partial X^\mu \bar{\partial} X_\mu + \psi^\mu \bar{\partial}\psi_\mu + \tilde{\psi}^\mu \partial \tilde{\psi}_\mu\right].$$

The appropriate boundary insertion for different brane systems is most easily understood through Chan-Paton factors at the string endpoints. For instance, in the brane-antibrane system the string endpoints couple to the superconnection [37, 38] and the boundary insertion is [15, 16]

$$e^{-S_{\partial\Sigma}} = \text{Tr}\, \boldsymbol{P} \exp\left[\int d\tau d\boldsymbol{\vartheta} \mathcal{M}(\boldsymbol{X})\right], \qquad \mathcal{M}(\boldsymbol{X}) = \begin{pmatrix} iA^1_\mu(\boldsymbol{X})\mathbf{D}_{\boldsymbol{\vartheta}}\boldsymbol{X}^\mu & \sqrt{\alpha'}\bar{T}(\boldsymbol{X}) \\ \sqrt{\alpha'}T(\boldsymbol{X}) & iA^2_\mu(\boldsymbol{X})\mathbf{D}_{\boldsymbol{\vartheta}}\boldsymbol{X}^\mu \end{pmatrix}. \tag{4.2}$$

Bold quantities denote superfields, and $\boldsymbol{P}$ denotes supersymmetric path ordering which is necessary to preserve worldsheet supersymmetry and gauge invariance. $A^{1,2}$ are the U(1) connections, and T is the tachyon charged under the relative U(1),

$$D_\mu T = \left[\partial + iA^-\right]_\mu T\,, \qquad \text{where} \qquad A^\pm_\mu = (A^1 \pm A^2)_\mu\,.$$

The diagonal entries of $\mathcal{M}$ are understood as the fields on the brane and antibrane, and the off-diagonal entries represent the fields associated with strings stretching between them. Equation (4.2) is written in one dimensional boundary superspace hence it is manifestly invariant under $\mathcal{N} = 1$ supersymmetry. The boundary superfields are (see appendix for conventions)

$$\boldsymbol{X}^\mu = X^\mu + i\boldsymbol{\vartheta}\sqrt{2\alpha'}\psi^\mu\,, \qquad \mathbf{D}_{\boldsymbol{\vartheta}} = \partial_{\boldsymbol{\vartheta}} + \boldsymbol{\vartheta}\partial_\tau\,.$$

The path ordered trace in (4.2) is most readily performed using a complex boundary fermion superfield $\boldsymbol{\Gamma} = \eta + \boldsymbol{\vartheta} F$, the quantization of which satisfies the algebra of the SU(2) matrices [39, 40]. Hence (4.2) can be written

$$e^{-S_{\partial\Sigma}} = \int \mathcal{D}\boldsymbol{\Gamma}\mathcal{D}\bar{\boldsymbol{\Gamma}} \exp\left[\int d\tau d\boldsymbol{\vartheta}\left(\boldsymbol{\Gamma}\mathbf{D}_{\boldsymbol{\vartheta}}\bar{\boldsymbol{\Gamma}} + \frac{i}{2}A^+_\mu(\boldsymbol{X})\mathbf{D}_{\boldsymbol{\vartheta}}\boldsymbol{X}^\mu + \sqrt{\alpha'}T\bar{\boldsymbol{\Gamma}} + \sqrt{\alpha'}\bar{T}\boldsymbol{\Gamma} + \right.\right.$$
$$\left.\left. + iA^-_\mu(\boldsymbol{X})\mathbf{D}_{\boldsymbol{\vartheta}}\boldsymbol{X}^\mu\bar{\boldsymbol{\Gamma}}\boldsymbol{\Gamma}\right)\right]\,. \tag{4.3}$$

When the gauge fields vanish, the boundary fermion superfields can be integrated to give

$$e^{-S_{\partial\Sigma}} = \exp\left[\alpha'\int d\tau\left(-T\bar{T} + 2\alpha'(\psi\cdot\partial T)\frac{1}{\partial_\tau}(\psi\cdot\partial\bar{T})\right)\right]\,. \tag{4.4}$$

The operator $1/\partial_\tau$ acting on a function $f(\tau)$ is defined to be the convolution of f with $\mathrm{sgn}(\tau)$ over the worldsheet boundary. With this insertion (4.1) remains Gaussian for a linear tachyon profile [15, 16]. The result can be interpreted as a spacetime action for the D$\bar{\text{D}}$ system. The path integral can still be performed when $A^+ \neq 0$, and given that the tachyon is charged under the relative U(1), there is a unique way to write the spacetime action in a gauge covariant form [17], giving

$$S_{\mathrm{D}\bar{\mathrm{D}}} = -\tau_9\int d^{10}x\, e^{-2\pi\alpha' T\bar{T}}\begin{bmatrix}\sqrt{-\det[G_1]}\,\mathcal{F}(\mathcal{X}_1+\sqrt{\mathcal{Y}_1})\,\mathcal{F}(\mathcal{X}_1-\sqrt{\mathcal{Y}_1})\\ +\sqrt{-\det[G_2]}\,\mathcal{F}(\mathcal{X}_2+\sqrt{\mathcal{Y}_2})\,\mathcal{F}(\mathcal{X}_2-\sqrt{\mathcal{Y}_2})\end{bmatrix},$$

$$(G_{\mu\nu})_{1,2} \equiv \left(g_{\mu\nu} + 2\pi\alpha' F^{1,2}_{\mu\nu}\right)\,, \qquad \begin{aligned}\mathcal{X}_{1,2} &\equiv 2\pi\alpha'^2 G^{\{\mu\nu\}}_{1,2} D_\mu T D_\nu \bar{T},\\ \mathcal{Y}_{1,2} &\equiv \left|2\pi\alpha'^2 G^{\mu\nu}_{1,2} D_\mu T D_\nu T\right|^2\,,\end{aligned}$$

where the function $\mathcal{F}(x)$ is the "boundary entropy" found in [14],

$$\mathcal{F}(x) = \frac{4^x x\Gamma(x)^2}{2\Gamma(2x)} = \frac{\sqrt{\pi}\Gamma(1+x)}{\Gamma(\frac{1}{2}+x)}\,.$$

As $x \to 0$, $\mathcal{F}(x) \to 1 + (2\ln 2)x$, yielding a tachyon mass $m_T^2 = -1/(4\ln 2)$. The factor of $2\ln 2$ is a well-known discrepancy between the BSFT tachyon mass and the worldsheet CFT mass.

5. BSFT for branes at angles

The most obvious way to extend the brane-antibrane BSFT formalism to a system of branes at an arbitrary angle is to T-dualize the boundary insertion (4.2) and turn on background values for the scalars representing the brane positions in the extra directions. However, this approach very quickly leads to non-Gaussian boundary interactions, and hence does not have a free-field solution. We find that an approach using boundary twist superfields is more fruitful, as was first performed in a small $(1-\nu)$, small $T\bar{T}$ expansion by [31] and was suggested in [41].

The twist superfield obtained in section 3 is

$$\boldsymbol{\mathfrak{T}}^{\pm} = \mathfrak{T}_0^{\pm} + i\boldsymbol{\vartheta}\mathfrak{T}_1^{\pm} = (\sigma s)^{\pm} - \frac{i}{\sqrt{2\alpha'}}\boldsymbol{\vartheta}(\mu u)^{\pm}\,, \qquad h_{\mathfrak{T}_0^{\pm}} = \frac{1}{2}\nu\,, \qquad h_{\mathfrak{T}_1^{\pm}} = \frac{1}{2}(\nu+1)\,,$$

where $\nu = \phi/\pi$, with ϕ being the angle between the branes. The superfields $\boldsymbol{\mathfrak{T}}^{+}$ and $\boldsymbol{\mathfrak{T}}^{-}$ differ in that they twist the bosons and fermions in opposite directions and must come in pairs to give a consistent worldsheet; i.e. the boundary conditions must flip from those of one D-brane to those of the other back and forth. The vertex superfield which interpolates between that of the D$\bar{\text{D}}$ tachyon and that of the DD gauge field was elucidated in section 3.4:

$$\begin{aligned}\mathcal{V}_{\mathfrak{T}^{+}} &= T\boldsymbol{\mathfrak{T}}^{+}e^{ik\cdot\boldsymbol{X}}\\ &= T\left[\mathfrak{T}_0^{+} + i\boldsymbol{\vartheta}\left(\mathfrak{T}_1^{+} - \sqrt{2\alpha'}\mathfrak{T}_0^{+}k\cdot\psi\right)\right]e^{ik\cdot X}\,,\end{aligned} \tag{5.1}$$

where X and ψ are in the untwisted directions only. The coupling T is to be interpreted as a tachyon with no momentum in the X^8 direction, yet for a full twist $\nu \to 1$, it is to be interpreted as the off-diagonal parts of $\frac{1}{\sqrt{2}}(A_8 - i\Phi)$, where Φ is the scalar representing the brane location in the transverse direction. A vertex operator $\mathcal{V}_{\mathfrak{T}^{-}}$ can be similarly constructed by flipping $\boldsymbol{\mathfrak{T}}^{+}$ and $\boldsymbol{\mathfrak{T}}^{-}$. Exponentiation of the correlation functions of these vertex operators leads to the sigma-model action for the lowest tachyonic state stretching between angled branes. Alternatively, the BSFT boundary insertion can be constructed as the direct generalisation of (4.2), by noting that only tachyon fields should carry twist superfields since they are the states stretching between branes, and that the twist superfields have the appropriate limiting forms in terms of the bosonic superfields. The boundary insertion should therefore be

$$e^{-S_{\partial\Sigma}} = \mathrm{Tr}\,\boldsymbol{P}\exp\left[\int d\tau d\boldsymbol{\vartheta}\mathcal{M}(\boldsymbol{X})\right], \qquad \mathcal{M}(\boldsymbol{X}) = \begin{pmatrix} 0 & \sqrt{\alpha'}\boldsymbol{\mathfrak{T}}^{-}\bar{T}(\boldsymbol{X}) \\ \sqrt{\alpha'}T(\boldsymbol{X})\boldsymbol{\mathfrak{T}}^{+} & 0\end{pmatrix},$$

where we have set the gauge fields corresponding to strings starting and ending on the same brane to zero for simplicity. We will comment on the inclusion of these fields in the Discussion section. This modification of the brane-antibrane worldsheet action reduces to the insertion for the D$\bar{\text{D}}$ system when $\nu = 0$ where $\boldsymbol{\mathfrak{T}}^{\pm}$ become $\mathbb{1}$. When $\nu = 1$, the twist superfields become $\sim \mathbf{D}_{\boldsymbol{\vartheta}}\boldsymbol{Z}^{(\dagger)}$, so the insertion is exactly that for the off-diagonal fields of the brane-brane system if T is interpreted as a combination of the gauge field in one direction and the transverse scalar. Note that the components of these off-diagonal gauge fields in the untwisted directions are not incorporated in this procedure because they correspond to massive string modes for all $\nu < 1$. The insertion will always produce a consistent worldsheet theory since the twist and anti-twist operators always appear together.

The worldsheet actions we write must be regarded as formal expressions since the treatment of the twist fields as classical fields in the action is ill-defined; these fields have fractional statistics so can be represented neither as complex numbers nor Grassman valued functions. We take these boundary actions to represent the appropriate formal sum of correlation functions, which are well defined for the twist fields. In evaluating partition sums with these boundary insertions, we will resort to correlation function methods in order to side-step any ambiguities. The formal action is used to determine the equations satisfied by the correlation functions in the interacting theories.

To proceed, what is usually done in the literature is to represent path ordered traces in terms of boundary fermions. Since we are working with operators of fractional statistics, the boundary fermion formalism is more intricate, so we will continue as far as possible without recourse to it. The supersymmetric path-ordering can be evaluated directly from the definition as written in [15]. We set the gauge fields to zero for simplicity and obtain

$$\begin{aligned}
\mathrm{Tr}\,\boldsymbol{P}\exp\int d\hat{\tau}\mathcal{M}(\hat{\tau}) &= \mathrm{Tr}\sum_{n=0}^{\infty}\int d\hat{\tau}_1\dots d\hat{\tau}_n\Theta(\hat{\tau}_{12})\Theta(\hat{\tau}_{23})\dots\Theta(\hat{\tau}_{n-1,n})\mathcal{M}(\hat{\tau}_1)\dots\mathcal{M}(\hat{\tau}_n)\\
&= \mathrm{Tr}\sum_{n=0}^{\infty}\int d\tau_1\dots d\tau_n\begin{bmatrix}\Theta(\tau_{12})\Theta(\tau_{23})\dots\Theta(\tau_{n-1,n})\times\\ \left[\mathcal{M}_1-\mathcal{M}_0^2\right](\tau_1)\dots\left[\mathcal{M}_1-\mathcal{M}_0^2\right](\tau_n)\end{bmatrix}\\
&= \mathrm{Tr}\,P\exp\int d\tau\left[\mathcal{M}_1(\tau)-\mathcal{M}_0^2(\tau)\right]\\
&= \mathrm{Tr}\,P\exp\left[\alpha'\int d\tau\begin{pmatrix}-\mathcal{T}_0^-\bar{T}T\mathcal{T}_0^+ & i\sqrt{2\alpha'}\mathcal{T}_0^-\partial_i\bar{T}\psi^i+i\mathcal{T}_1^-\bar{T}\\ i\sqrt{2\alpha'}\partial_jT\psi^j\mathcal{T}_0^++iT\mathcal{T}_1^+ & -T\mathcal{T}_0^+\mathcal{T}_0^-\bar{T}\end{pmatrix}\right],
\end{aligned}\qquad(5.2)$$

in which $\hat{\tau}_{12}=\tau_1-\tau_2+\boldsymbol{\vartheta}_1\boldsymbol{\vartheta}_2$, P in the result is standard path ordering after integration over superspace, and $\mathcal{M}_{0,1}$ are the parts of the matrix $\mathcal{M}$ which are proportional to zero and one power of $\boldsymbol{\vartheta}$ respectively. The path ordered trace is most simple to evaluate using boundary fermions, but in the present case cocycles are needed because the twist fields have fractional statistics. Proceeding by assuming such cocycles can be defined, we write the simplified boundary insertion after taking the path ordered trace by modified boundary fermions as

$$e^{-S_{\partial\Sigma}}=\exp\int\alpha' d\tau\left[-\mathcal{T}_0^-\bar{T}T\mathcal{T}_0^++\left(\sqrt{2\alpha'}\mathcal{T}_0^-\partial\bar{T}\cdot\psi+\mathcal{T}_1^-\bar{T}\right)\frac{1}{\partial_\tau}\left(\sqrt{2\alpha'}\partial T\cdot\psi\mathcal{T}_0^++T\mathcal{T}_1^+\right)\right].\qquad(5.3)$$

The T constant version of this worldsheet action was derived in [31] using boundary fermions, but that derivation assumed $\mathcal{T}_0$ and $\mathcal{T}_1$ are always bosonic and fermionic respectively. Alternatively we can construct this formal insertion as the simplest expression which satisfies the necessary properties that

- it exhibits explicit $\mathcal{N}=1$ 1D SUSY,
- boundary condition changing operators always appear in $\mathcal{T}^-\mathcal{T}^+$ pairs, for a consistent worldsheet. Although individual twist fields are fractionally moded, the $\mathcal{T}^-\mathcal{T}^+$ pairs behave as scalars.
- in the limit $\nu\to 0$, because we have $\mathcal{T}_0\to\mathbb{1}$ and $\mathcal{T}_1\to 0$, the D$\bar{\text{D}}$ boundary insertion for the tachyon is exactly reproduced,
- for $\nu\to 1$, the boundary action becomes that for gauge fields and scalars on the brane-brane system. For instance, when ∂T is set to zero and $\nu=1$ we have

$$S_{\partial\Sigma}\to\int d\tau\left(-\frac{i}{2}T\bar{T}\right)\left[X^8(X^9)'-2\alpha'\psi^8\psi^9\right].$$

If we relabel the coupling $T\bar{T} \to i(\bar{\Phi}A_8 - \bar{A}_8\Phi)$ (which is real, with A_8 and Φ the complex off-diagonal parts of the U(2) gauge field and brane scalar), this is exactly the correct boundary insertion for the brane brane system after setting all field derivatives and all other fields in the system to zero; i.e. this is just the insertion for that part of $D\Phi$ which we have not set to zero, and in this case the path ordered trace is trivial. All other fields were set to zero since they are massive for $\nu < 1$, or because we neglected field derivatives.

As such, we treat (5.3) as a well-motivated definition of the boundary insertion for the tachyonic state for branes at angles, even though we are unable to give a thorough derivation of it.

5.1 Tachyon potential

The tachyon potential is obtained by inserting $T(x) = T$ (constant) on the boundary. The non-trivial ν dependence is completely given by the twist fields. The potential is

$$V_\nu(T\bar{T}) = \int \mathcal{D}X\mathcal{D}\psi\, e^{-S_\Sigma} \exp\int d\tau(-\alpha' T\bar{T})\left[\mathcal{T}_0^-\mathcal{T}_0^+ - \mathcal{T}_1^-\frac{1}{\partial_\tau}\mathcal{T}_1^+\right]. \tag{5.4}$$

Because of the ambiguity of treating the twist fields as classical worldsheet fields, the path integral cannot be performed directly. Since all twist field correlators are well defined however, this potential can be evaluated by identifying the equations they satisfy in the presence of the boundary insertion. Writing $T\bar{T} = y$, we see that

$$\begin{aligned}\partial_y \ln V_\nu(y) &= -\alpha' \int d\tau \left\{\frac{\langle \mathcal{T}_0^-(\tau)\mathcal{T}_0^+(\tau)\rangle_y}{\langle 1\rangle_y} - \frac{\left\langle \mathcal{T}_1^-(\tau)\frac{1}{\partial_\tau}\mathcal{T}_1^+(\tau)\right\rangle_y}{\langle 1\rangle_y}\right\} \\ &= -\frac{\alpha'}{2}\int d\tau\left\{G^-(0;y) + G^+(0;y) + \frac{1}{\partial_\epsilon}\left[H^-(0;y) + H^+(0;y)\right]\right\}. \end{aligned}\tag{5.5}$$

The y-dependent correlators are defined as

$$\langle \mathcal{O}\rangle_y \equiv \left\langle \mathcal{O}\exp\int d\tau(-\alpha' y)\left[\mathcal{T}_0^-\mathcal{T}_0^+ - \mathcal{T}_1^-\frac{1}{\partial_\tau}\mathcal{T}_1^+\right]\right\rangle,$$

so $V_\nu(y) = \langle 1\rangle_y$, and we have defined the point-split correlators of twist fields

$$G^\mp(\epsilon; y) \equiv \frac{\langle \mathcal{T}_0^\mp(\epsilon)\mathcal{T}_0^\pm(0)\rangle_y}{\langle 1\rangle_y}, \qquad H^\mp(\epsilon; y) \equiv \frac{\langle \mathcal{T}_1^\mp(\epsilon)\mathcal{T}_1^\pm(0)\rangle_y}{\langle 1\rangle_y}, \tag{5.6}$$

with $\epsilon \to 0^+$. As in [14, 16] we shall use a point-splitting method to evaluate the action.[4] By differentiating (5.6), the ordered Green's functions are easily seen to satisfy the differential

[4] One can see that equation (5.5) involves both orderings as the limit $\epsilon \to 0^+$ is taken by looking at the original regularization prescription adopted in [14, 16] and noting that our fields are complexified; i.e. $\langle X^8(\epsilon)X^8(0) + X^9(\epsilon)X^9(0)\rangle = \langle Z(\epsilon)Z^\dagger(0) + Z^\dagger(\epsilon)Z(0)\rangle$.

equations

$$\partial_y G^{\mp}(\epsilon; y) = -\alpha' \int d\tau\, G^{\pm}(\tau - \epsilon; y) G^{\mp}(\tau; y)\,,$$
$$\partial_y H^{\mp}(\epsilon; y) = -\alpha' \frac{1}{\partial_\epsilon} \int d\tau H^{\pm}(\tau - \epsilon; y) H^{\mp}(\tau; y)\,. \tag{5.7}$$

A problem with these equations is that they are not valid for the D$\bar{\text{D}}$ case, $\nu = 0$, for which $\langle 1 \rangle_y \sim \exp(-2\pi\alpha' y)$ and $G^{\pm}(y) = 1$. This will cause a discontinuity in the ν-dependent solution. However, the cause of the discontinuity is well understood - the operators $\mathcal{T}^{\mp}\mathcal{T}^{\pm}$ merge in the limit $\nu \to 0$, and so (5.7) includes too many Wick contractions only for that case. Also we find $\lim_{\nu\to 0} V_\nu(y)$ is not very different from the D$\bar{\text{D}}$ potential, and leads to the same physical behaviour.

These equations can be easily solved if one expands the Green's functions of the interacting theory $G^{\pm}$ and $H^{\pm}$ in frequency modes in a manner consistent with the ordering prescription adopted earlier. In particular, G^{-} and H^{-} are expanded in positive modes, while G^{+} and H^{+} are negatively moded. The *ansatz* for the correlators is

$$G^{\mp}(\epsilon; y) = \sum_{n=0}^{\infty} \mathcal{G}_n^{\mp}(y) e^{\pm i\epsilon[n+\frac{1}{2}\nu]}\,, \qquad H^{\mp}(\epsilon; y) = \sum_{n=0}^{\infty} \mathcal{H}_n^{\mp}(y) e^{\pm i\epsilon[n+\frac{1}{2}(\nu+1)]}\,.$$

The equations which the mode coefficients satisfy, and their boundary conditions from the free theory, $y = 0$, correlators of section 3.3 are

$$\frac{d}{dy}\mathcal{G}_n^{-}(y) = \frac{d}{dy}\mathcal{G}_n^{+}(y) = -2\pi\alpha' \mathcal{G}_n^{+}(y)\mathcal{G}_n^{-}(y)\,, \qquad \mathcal{G}_n^{\mp}(0) = \frac{e^{\mp i\frac{\pi}{2}\nu}}{n\,\mathrm{B}(n,\nu)}\,,$$
$$\frac{d}{dy}\mathcal{H}_n^{-}(y) = -\frac{d}{dy}\mathcal{H}_n^{+}(y) = \frac{2\pi\alpha' i}{n + \frac{1}{2}(\nu+1)}\mathcal{H}_n^{+}(y)\mathcal{H}_n^{-}(y)\,, \qquad \mathcal{H}_n^{\mp}(0) = \frac{\mp i e^{\mp i\frac{\pi}{2}\nu}}{\mathrm{B}(n+1,\nu)}\,, \tag{5.8}$$

which have solutions

$$\mathcal{G}_n^{\mp}(y) = \frac{\sin(\frac{\pi\nu}{2})}{n\,\mathrm{B}(n,\nu)} \left[\cot\left(\frac{\sin(\frac{\pi\nu}{2})}{n\,\mathrm{B}(n,\nu)} 2\pi\alpha' y + \tfrac{\pi\nu}{2}\right) \mp i\right],$$
$$\mathcal{H}_n^{\mp}(y) = -\frac{\sin(\frac{\pi\nu}{2})}{\mathrm{B}(n+1,\nu)} \left[1 \pm i\cot\left(\frac{\sin(\frac{\pi\nu}{2})}{[n+\frac{1}{2}(\nu+1)]\,\mathrm{B}(n+1,\nu)} 2\pi\alpha' y + \tfrac{\pi\nu}{2}\right)\right]. \tag{5.9}$$

Finally, we can reconstruct the potential using (5.5) and these solutions.

$$V_\nu(y) = N_\nu \prod_{n=0}^{\infty} \frac{\sin\left(\dfrac{\sin(\frac{\pi\nu}{2}) 2\pi\alpha' y}{[n+\frac{1}{2}(\nu+1)]\,\mathrm{B}(n+1,\nu)} + \dfrac{\pi\nu}{2}\right)}{\sin\left(\dfrac{\sin(\frac{\pi\nu}{2})}{n\,\mathrm{B}(n,\nu)} 2\pi\alpha' y + \dfrac{\pi\nu}{2}\right)}. \tag{5.10}$$

N_ν is the unknown normalization which we shall discuss in section 5.3. Note that $V_\nu(0) = N_\nu[\sin(\frac{\pi\nu}{2})]^{-\frac{1}{2}}$, where ζ-function regularization must be used to evaluate the product at $y = 0$. (5.10) leads to the D$\bar{\text{D}}$ and DD potentials,

$$V_0(T) = \frac{V_0(0)}{1 + 2\pi\alpha' T\bar{T}}\,, \qquad V_1(T) = \frac{V_1(0)}{\sqrt{\cos(2\pi\alpha' T\bar{T})}}\,.$$

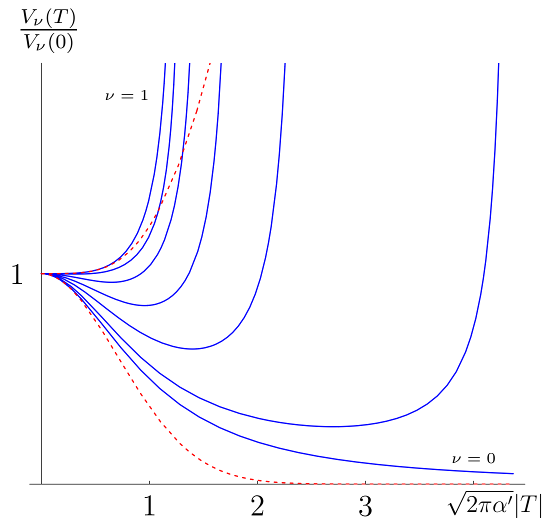

Figure 4: Solid curves show $V_\nu(T)$ for $\nu = 0, 0.1, 0.3, 0.5, 0.7, 0.9$ and 1. The dotted curves show the gaussian $\mathrm{D\bar{D}}$ tachyon potential and the DBI, the expected results for $\nu = 0$ and $\nu = 1$.

In the $\nu = 0$ limit only the $n = 0$ term from the product has y dependence. The reason for the discrepancy from the expected tachyon potential, $\exp(-2\pi\alpha' T\bar{T})$, was explained previously; these two functions have similar behaviour so we do not consider this discontinuity a serious problem. Naïvely the $\nu = 1$ result is constant since the numerator and denominator at each n are identical, but we must take into account that the numerator comes from worldsheet modes of frequencies $\mathbb{Z} + \frac{\nu+1}{2}$, while the denominator from modes of frequencies $\mathbb{Z} + \nu/2$; using ζ-function regularization as in the derivation of the DBI [42],

$$\frac{V_1(T)}{V_1(0)} = \frac{\prod_{m=1}^{\infty} \cos(2\pi\alpha' T\bar{T})}{\prod_{r=\frac{1}{2}}^{\infty} \cos(2\pi\alpha' T\bar{T})} = \left[\cos(2\pi\alpha' T\bar{T})\right]^{\zeta(0,1)-\zeta(0,1/2)} = \left[\cos(2\pi\alpha' T\bar{T})\right]^{-1/2} .$$

That this differs from the known result, $\sim \sqrt{1 + \frac{1}{4}(2\pi\alpha' T\bar{T})^2}$ is not yet understood, but again the two potentials shall give similar physical behaviour. The regularization by ζ-function methods means that we must take infinitely many terms into account for the $\nu = 1$ case; as $\nu : 0 \to 1$ more and more terms in the product (5.10) become relevant. This can also be seen by considering that the frequencies of the sine functions for higher terms are less than those for lower n, so the higher n terms tend to be constant over the range of physical interest. As ν approaches 1, the frequencies of all terms approach one value, so the higher terms vary more and more. Note also that a tower of (infinite number of) massive states becomes massless at $\nu = 1$. They are not included in the above calculation.

Figure 4 shows that for intermediate values of ν, the potential has appropriate properties for branes at angles. Firstly, we see that the minimum of V_ν is at $T \to \infty$ for $\nu = 0$, and smoothly moves to finite values for intermediate ν, to approach $T = 0$ at $\nu = 1$. The potential also approaches a singularity at $2\pi\alpha' T\bar{T} = \pi(1 - \frac{\nu}{2})/\sin\frac{\pi\nu}{2}$. This is not problematic given that the tachyonic state is localized about the intersection and the tachyon condensation must end at some finite expectation value, as will be discussed in section 5.3.

That the potential is an infinite product of poles and zeros can be disconcerting, but it is always well-behaved in the region of physical interest because the frequencies of the higher modes decrease - so the potential always reaches the singularity from the $n = 0$ term sooner than it reaches any other zero or pole.

The mass term can also be obtained from (5.10):

$$\begin{aligned}\tilde{m}^2_{\mathcal{T}}(\nu) \equiv \partial_y V_\nu(y)|_{y=0} &= 2\pi\alpha' \cos\frac{\pi\nu}{2}\sum_{n=0}^{\infty}\frac{\Gamma(n+\nu)}{\Gamma(n+1)\Gamma(\nu)}\left[\frac{n+\nu}{n+\frac{1}{2}(\nu+1)}-1\right]\\ &= -\pi\alpha'\cos\frac{\pi\nu}{2}(1-\nu)\frac{\Gamma(1-\nu)\Gamma(\frac{1+\nu}{2})}{\Gamma(\frac{3-\nu}{2})}\,. \end{aligned} \tag{5.11}$$

This is always negative and vanishes at $\nu = 1$, as can be seen in figure 4. It is not the physical mass of the twisted tachyon state however, since the kinetic term is not canonically normalized; in the section to follow, we calculate the normalization of the kinetic term to extract the physical tachyon mass.

5.2 Tachyon kinetic terms

Unlike in the brane-antibrane case, the tachyon potential is the only quantity which can be evaluated exactly using BSFT. This is because in order to angle the one brane with respect to the other, additional dynamical worldsheet fields need to be included in the action (5.3), and this means that a linear tachyon profile corresponds to a non-Gaussian path integration. Since the coupling of the linear tachyon profile corresponds in the low energy action to the tachyon kinetic term, the complete kinetic term cannot be obtained for branes at angles. We can, however, calculate the tachyon mass by performing an expansion of (5.3) in $\partial_i T$; the coefficient of the term of $(\partial_i T)^2$ then gives the normalization of the kinetic term in the low energy effective action, which shall allow us to determine the mass of the lowest tachyon.

With T linear in one of the un-angled directions along the brane, $T = uX$, the coefficient of $-2\pi\alpha' u^2$ in (5.3) is

$$\begin{aligned}&\left\langle \mathcal{T}_0^- X(\tau_1) X \mathcal{T}_0^+(\tau_2)\right\rangle - \frac{1}{\partial_{\tau_2}}\left\langle \left[\sqrt{2\alpha'}\mathcal{T}_0^-\psi + \mathcal{T}_1^- X\right](\tau_1)\left[\sqrt{2\alpha'}\psi\mathcal{T}_0^+ + X\mathcal{T}_1^+\right](\tau_2)\right\rangle = \\ &= \underbrace{\left\langle \mathcal{T}_0^-\mathcal{T}_0^+\right\rangle(\epsilon)\left\langle XX\right\rangle(\epsilon)}_{2\alpha'\mathcal{C}_0(\epsilon)} + \frac{1}{\partial_\epsilon}\left[2\alpha'\underbrace{\left\langle \mathcal{T}_0^-\mathcal{T}_0^+\right\rangle(\epsilon)\left\langle\psi\psi\right\rangle(\epsilon)}_{\mathcal{C}_1(\epsilon)} + \underbrace{\left\langle \mathcal{T}_1^-\mathcal{T}_1^+\right\rangle(\epsilon)\left\langle XX\right\rangle(\epsilon)}_{2\alpha'\mathcal{C}_2(\epsilon)}\right],\end{aligned}$$

where $\epsilon = \tau_1 - \tau_2 \to 0$. The ν-dependent coefficient of the kinetic term for the tachyon will therefore be given by

$$\mathcal{K}_\nu \equiv -4\pi\alpha'^2\left\{\mathcal{C}_0(0) + \frac{1}{\partial_\epsilon}\left[\mathcal{C}_1 + \mathcal{C}_2\right](0)\right\}. \tag{5.12}$$

To calculate this quantity we expand the correlators in modes, and set $\epsilon = 0$. The individual sums obtained diverge but the combination is finite. Formally, in order to obtain the mode expansions, we add a small negative imaginary component to ϵ. As in the previous section, we average over the two physically distinct point-splitting schemes, giving both positive and negative frequency modes.

$$\begin{aligned}
\mathcal{C}_0(\epsilon) &= \frac{-\ln\left(2\sin\frac{\epsilon}{2}\right)}{\left[2\sin\frac{\epsilon}{2}\right]^{\nu}} = \frac{1}{2}\sum_{\substack{m=1\\n=0}}^{\infty} \frac{e^{-i\frac{\pi}{2}\nu}e^{i\epsilon[n+m+\frac{1}{2}\nu]} + e^{i\frac{\pi}{2}\nu}e^{-i\epsilon[n+m+\frac{1}{2}\nu]}}{mn\,\mathrm{B}(n,\nu)}\,,\\
\mathcal{C}_1(\epsilon) &= \frac{1}{\left[2\sin\frac{\epsilon}{2}\right]^{\nu+1}} = \frac{1}{2}\sum_{\substack{m=0\\n=0}}^{\infty} \frac{e^{-i\frac{\pi}{2}(\nu+1)}e^{i\epsilon[n+m+\frac{1}{2}(\nu+1)]} + e^{i\frac{\pi}{2}(\nu+1)}e^{-i\epsilon[n+m+\frac{1}{2}(\nu+1)]}}{n\,\mathrm{B}(n,\nu)}\,,\\
\mathcal{C}_2(\epsilon) &= \frac{-\nu\ln\left(2\sin\frac{\epsilon}{2}\right)}{\left[2\sin\frac{\epsilon}{2}\right]^{\nu+1}} = \frac{\nu}{2}\sum_{\substack{m=1\\n=0}}^{\infty} \frac{e^{-i\frac{\pi}{2}(\nu+1)}e^{i\epsilon[n+m+\frac{1}{2}(\nu+1)]} + e^{i\frac{\pi}{2}(\nu+1)}e^{-i\epsilon[n+m+\frac{1}{2}(\nu+1)]}}{n\,\mathrm{B}(n,\nu+1)}\,.
\end{aligned}$$

The required combinations are the individually divergent sums

$$\begin{aligned}
\mathcal{C}_0(0) &= \cos\left(\frac{\pi\nu}{2}\right)\sum_{\substack{m=1\\n=0}}^{\infty}\frac{\Gamma(n+\nu)}{m\Gamma(\nu)\Gamma(n+1)}\,,\\
\frac{1}{\partial_\epsilon}\mathcal{C}_1(0) &= -\cos\left(\frac{\pi\nu}{2}\right)\sum_{\substack{m=0\\n=0}}^{\infty}\frac{\Gamma(n+\nu)}{[n+m+\frac{1}{2}(\nu+1)]\Gamma(\nu)\Gamma(n+1)}\,,\\
\frac{1}{\partial_\epsilon}\mathcal{C}_2(0) &= -\cos\left(\frac{\pi\nu}{2}\right)\sum_{\substack{m=1\\n=0}}^{\infty}\frac{\Gamma(n+\nu+1)}{m[n+m+\frac{1}{2}(\nu+1)]\Gamma(\nu)\Gamma(n+1)}\,.
\end{aligned}$$

The normalization of the kinetic term (5.12) is obtained by performing the subtractions. The then convergent sums can be re-expressed as convergent integrals:

$$\begin{aligned}
\mathcal{K}_\nu &= -4\pi\alpha'^2\cos\left(\frac{\pi\nu}{2}\right)\sum_{n=0}^{\infty}\frac{\Gamma(n+\nu)}{\Gamma(\nu)\Gamma(n+1)}\left\{-\frac{1}{n+\frac{1}{2}(\nu+1)}+\sum_{m=1}^{\infty}\frac{\frac{1}{2}(1-\nu)}{m[n+m+\frac{1}{2}(\nu+1)]}\right\}\\
&= -4\pi\alpha'^2\cos\left(\frac{\pi\nu}{2}\right)\int_0^1 dx\,(1-x)^{-\nu}x^{\frac{1}{2}(\nu-1)}\left\{-1-\frac{1}{2}(1-\nu)\ln(1-x)\right\}.
\end{aligned}$$

Finally, the integrals are expressible in terms of gamma and Polygamma functions $\Psi(y) = \frac{d}{dy}\ln\Gamma(y)$,

$$\begin{aligned}
\mathcal{K}_\nu &= -4\pi\alpha'^2\cos\left(\tfrac{\pi\nu}{2}\right)\frac{\Gamma(1-\nu)\Gamma(\frac{1+\nu}{2})}{\Gamma(\frac{3-\nu}{2})}\left\{-1+\frac{1}{2}(1-\nu)\left[\Psi(\tfrac{3-\nu}{2})-\Psi(1-\nu)\right]\right\}\\
&= 2\pi\alpha'^2\frac{\sqrt{\pi}\Gamma(1-\frac{\nu}{2})}{2^\nu\Gamma(\frac{3-\nu}{2})}\left\{1+(1-\nu)\left[\Psi(2-\nu)-\Psi(\tfrac{3-\nu}{2})\right]\right\}. \qquad (5.13)
\end{aligned}$$

In the last line Γ and Ψ-function identities have been used to write the result in a form which shows that $\mathcal{K}_\nu$ is finite and non-zero for $\nu\in[0,1]$. Using this normalization for the kinetic term and the non-normalized mass (5.11) obtained from the tachyon potential, the ν-dependent mass is shown in figure 5. The tachyon mass at $\nu=0$ agrees with that in the D$\bar{\text{D}}$ system in BSFT.

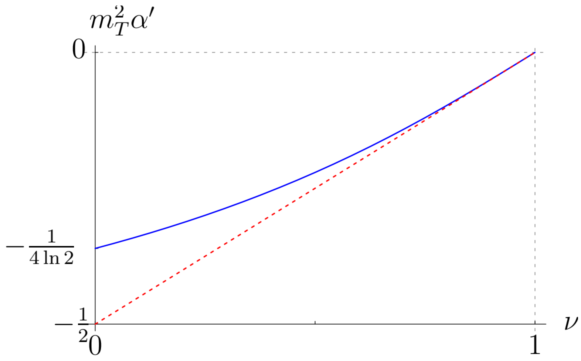

Figure 5: The ν-dependent tachyon mass calculated via BSFT (solid curve) and that mass calculated from worldsheet CFT (dotted line).

5.3 Low energy effective action

The low energy effective action for the tachyon between two D8-branes at angles, for which the tachyon is localized about the intersection and has only kinetic terms in the un-angled directions is given by

$$S(T) = \int d^8x \; V_\nu(T)\sqrt{-\mathrm{P}[g]}\,\left(1 + \mathcal{K}_\nu\; \partial_a T \partial^a \bar{T}\right) + \cdots\,, \tag{5.14}$$

where 'a' runs over spacetime directions transverse to Z and $Z^\dagger$. The normalization of the action, which measures brane tension and which we will now fix, is factored into $V_\nu(T)$.

It is clear from energetic considerations that the strings stretched between two angled branes are localized about the intersection point. It is also obvious that at the special angles of $\nu = 0$ and 1 those strings are no longer localized. The region of string localization must cover the entire system for $\nu = 0$, reduces to a slight region about the intersection, and increase again beyond $\nu = 1/2$ to once more cover the entire system at $\nu = 1$. The localization must also be symmetric about $\nu = 1/2$ because this argument does not depend on brane charge. Localization is also strongly suggested by the spectrum, as in figure 3 and the discussion following it.

We can estimate the size of the region of localization as a function of ν from a simple geometric argument (see figure 1). For a brane configuration with $\nu < 1/2$, the geometric volume of interest along the brane will be $v_1 \sim [\sin(\frac{\pi\nu}{2})]^{-1}$. For $\nu > 1/2$, this length is $v_2 \sim [\cos(\frac{\pi\nu}{2})]^{-1}$. To produce an estimate which possesses the required symmetry about $\nu = 1/2$, the localization volume must be an average of these two lengths, and we find that the geometric average $\sqrt{v_1 v_2} \sim [\cos(\frac{\pi\nu}{2})\sin(\frac{\pi\nu}{2})]^{-1/2}$ matches well the result that was calculated from a low energy analysis in [34].

We expect that $V_\nu(0)$ is proportional to the volume along the brane where the tachyon is localized. This is sensible because we see from figure 4 that for intermediate values of ν, the relative potential has minimum at $0 \le V_\nu(T)/V_\nu(0) \le 1$; physically, because only a small part of the branes play a role in recombination, if the potential were proportional to the infinite brane volume (as in the DD and D$\bar{\text{D}}$ cases), there would be no change in the relative energy during recombination (except when $\nu = 0$) and the minimum of the potential would not change smoothly as it does. The potential must therefore be proportional only to a finite part of the brane volume, and it is sensible to assume that that part is the volume of tachyon localization. By matching $V_\nu(0)$ with the energy contained in the region of localization, the normalization of the potential, N_ν of (5.10), is determined to be

$$N_\nu = 2\tau_8 \sqrt{\frac{2\pi\alpha'}{\cos(\frac{\pi\nu}{2})}}\,, \qquad \Longrightarrow V_\nu(0) = 2\tau_8 \sqrt{\frac{2\pi\alpha'}{\cos(\frac{\pi\nu}{2})\sin(\frac{\pi\nu}{2})}}\,. \tag{5.15}$$

Furthermore, this argument implies that the X^8 direction has already been integrated out in the derivation of the potential, so the low energy effective action is an action for dynamics in the 8 remaining spacetime dimensions of the 9-dimensional brane world volume.

In the boundary insertion evaluated above, we consider a tachyon field with dependence on the transverse coordinates $\boldsymbol{X}$. We have in mind the lowest lying tachyonic state of the spectrum $|0\rangle_\nu$. The dynamics also involves operators of higher conformal weights, some of which will be relevant operators (depending on how close we are to $\nu = 0$ as argued at the end of section 2); others will correspond to massive modes or irrelevant perturbations. In this work, we truncate to the cross-section (in worldsheet coupling space) corresponding to a single complex tachyon field. Furthermore, it is easy to see from energetics that all string modes stretched between the branes would be localized near the brane intersection point. Therefore, from our BSFT formalism, the brane worldvolume accessible to these strings is transverse to the angling plane Z-$Z^\dagger$. It is however sometimes useful — in particular when $\nu \sim 0$ or $\nu \sim 1$ — to think of a more general tachyon field with a profile with respect to say $Z + Z^\dagger$, with this combination of Z and $Z^\dagger$ being considered as part of the worldvolume of the brane system. To describe the dynamics in these variables, we may write a new field $T(X, Z, Z^\dagger) = \sum_{n=0} T_n(X) f_n(Z + Z^\dagger)$ with the expansion modes corresponding to the states $(\alpha^\dagger_{-\nu})^n\,|0\rangle_\nu$, with conformal dimension $((2n+1)\nu - 1)/2$. Unlike standard toroidal Kaluza Klein compactification, the complete set given by $f_n(Z + Z^\dagger)$ is necessarily a set of functions localized about $Z = 0$. For example, it was shown in [34] through a low energy treatment at small angles, that the lowest mode $f_0(Z+Z^\dagger)$ is a Gaussian centered at $Z = 0$. The action for the spacetime field $T(X, Z, Z^\dagger)$ would also have an additional integral along the worldvolume direction $Z + Z^\dagger$. However, in our formalism, we obtain by construction an action with this worldvolume direction already integrated out.

Truncating to the lowest tachyon field, we are choosing to perturb with the simpler profile $T(X, Z, Z^\dagger) = T_0(X) f_0(Z + Z^\dagger)$ to capture part of the dynamics. Normalizing the kinetic term of the new tachyon field canonically, we can argue that the field $T_0(X)$ is proportional to our $T(X)$ after integrating out the new worldvolume direction; this naturally can change the form of the potential between the two pictures. What matters however is the effective potential written in terms of $T(X)$. $T(X)$ may then be viewed as an average measure of the separation between the branes - averaged over the profile in the $Z + Z^\dagger$ direction. More conveniently and with the appropriate rescaling, we may think of it as $T(X, Z = 0)$; i.e. the separation between the branes at their point of nearest proximity. This leads us to believe that our perturbation tracks the dynamics of the more

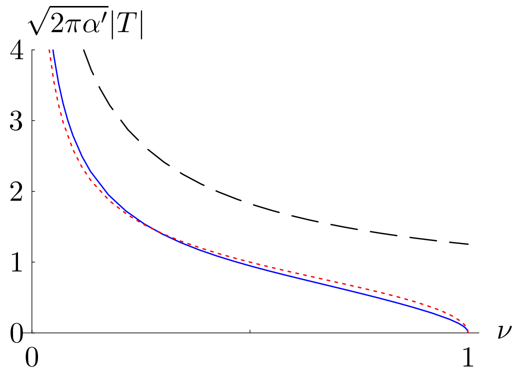

Figure 6: The value of $|T|$ for which $V_\nu(T)$ is minimal (solid), the approximation to it, $\sqrt{2\pi\alpha'}\langle|T|\rangle = \sqrt{\cot(\frac{\pi\nu}{2})}$ (dotted), and the critical value of $|T|$ for which the potential reaches a pole (dashed).

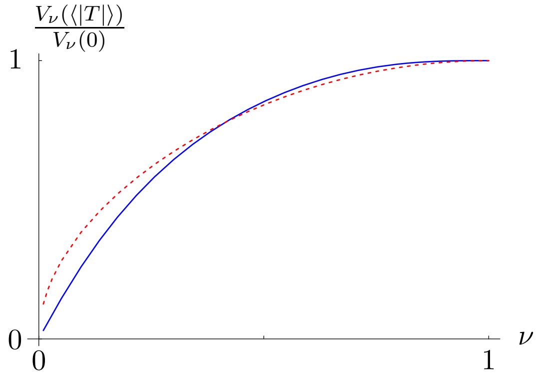

Figure 7: The value of the relative potential at its minimum (solid) and the approximation to it $\sim \sqrt{\sin(\frac{\pi\nu}{2})}$ (dotted), as functions of ν.

general tachyon field to leading order in resolution in Z and $Z^\dagger$. To resolve more of the dynamics in the twisted directions, we would then need to insert in the partition function vertex operators for the higher modes of the string. Note that a similar tower of massive states becomes massless at $\nu = 1$.

6. Brane recombination

In this section, we present a preliminary analysis of the physics encoded in the tachyon potential (5.10). We leave a more detailed study of the recombination mechanism to future work.

As argued earlier, we expect that the strings stretched between two angled branes are necessarily localized at the intersection point for $\nu \neq 0, 1$. In the formalism we have adopted, we have perturbed the boundary of the worldsheet with the lowest lying tachyon state - corresponding to, in the language of [34], a Gaussian tachyon profile along the branes in the angling plane with spread size given by $\sim [\cos(\frac{\pi\nu}{2})\sin(\frac{\pi\nu}{2})]^{-1/2}$. As argued in the previous section, our boundary insertion may be thought of as tracking the magnitude of this Gaussian. In this context, the constant tachyon value is physically the separation distance between the branes at their point of closest proximity as shown in figure 1. This is clear near $\nu = 1$ where the tachyon is explicitly some linear combination of the off-diagonal scalars representing brane separation and the off-diagonal gauge field; we assume this correspondence holds for all angles.

An interesting novel feature of our potential is that, for generic ν, the potential has a minimum for a finite expectation value of the tachyon. This suggests that the recombined branes separate up to a fixed extent as the tachyon condenses. The resultant configuration will then be two bent branes separated by more than a string length and so not interacting by open string modes. The configuration will clearly evolve from that stage; classically, and to this level of approximation, there will be oscillation about this configuration of bent branes (if the system is compactified), yet at this new vacuum, the original tachyon field has become stable. In [34], it was argued that the small angle analysis at low energies is valid until the tachyon reaches an expectation value of $\langle|T|\rangle \sim \sqrt{\cot(\frac{\pi\nu}{2})}$; beyond this point, the shape of the recombined branes begins to kink and the leading low energy approximation breaks down. At this $\langle|T|\rangle$, energy loss can be calculated by integrating the shape of the recombined brane. This gives as the relative energy remaining $\sim \sqrt{\sin(\frac{\pi\nu}{2})}$. Note that this is the square root of the same ratio when the final state is two straight parallel branes, giving credence to the interpretation of the final state as bent branes.

We can calculate these quantities numerically from our potential (5.10) assuming that $V_\nu(T)$ is proportional only to that part of the brane which plays some role in the recombination; i.e. $V_\nu(0)$ is proportional to the volume along the brane in which the tachyon is localized. The results are shown in figures 6 and 7. We find that the numerical values of the stable tachyon expectation value and the potential at its minimum are to an excellent approximation given by

$$\sqrt{2\pi\alpha'}\,\langle|T|\rangle \sim \sqrt{\cot\left(\frac{\pi\nu}{2}\right)}, \qquad \frac{V_\nu(\langle|T|\rangle)}{V_\nu(0)} \sim \sqrt{\sin\left(\frac{\pi\nu}{2}\right)}, \tag{6.1}$$

in good agreement with the ν-dependence of the low energy estimates of these quantities at the regime where the low energy approximation breaks down.

To summarize the interpretation of these results, the fact that $V_\nu(T)$ has a minimum at finite $\langle|T|\rangle$ suggests that the tachyonic mode is localized about the intersection point, as expected from other physical considerations, and that the potential is proportional to the localization volume. This tachyonic mode drives the dynamics of the brane recombination to a certain separation, beyond which the system evolves as two bent branes with only massive open string (and closed strings) interactions between them.

7. Discussion

In this work, we have developed the formalism of twist fields, which are frequently employed in the study of orbifolds, to the BSFT of angled branes. It was shown that the twist fields initiate worldsheet spectral flow from the NS$-$ spectrum on the D$\bar{\text{D}}$ system to the NS$+$ spectrum on the DD. The tachyon potential (5.10), derived exactly from this formalism, has all the correct properties to describe the recombination of intersecting branes.

We have calculated the canonical kinetic term for the tachyon, but because the twist fields intertwine with the worldsheet degrees of freedom, the complete dependence of the action on tachyon first derivatives cannot be obtained by our methods. There will therefore be $\mathcal{O}(\partial T)^4$ corrections to the low energy effective action (5.14), in addition to higher derivative corrections. The study of vortices for a system of angled branes has many important applications [8, 13], but a thorough understanding of vortices requires higher kinetic terms. For instance, in order to study the lower dimensional D-branes as solitons in the D$\bar{\text{D}}$ system it is necessary to know the action to all powers of ∂T [15, 16]. While the complete expression for the kinetic term requires solving an interacting worldsheet theory, it may be possible to simply deduce the $\partial T \to \infty$ behaviour directly from the worldsheet boundary action.

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Another issue of future interest is the inclusion of gauge fields and scalars on the diagonal of the boundary perturbation matrix. Transverse gauge fields may be included by covariantizing the spacetime derivatives in the low energy effective action. In the twisted directions, we may insert scalar/gauge field perturbations on the worldsheet boundary directly. In the simplest scenario, their inclusion would modify the boundary interaction in (5.3) by adding terms of the form $\sim \Phi X' + A\dot{X}+$(derivatives), Φ, A being scalar and gauge fields in the twisted plane. Typically, this modification makes the worldsheet theory interacting and is more difficult to handle. Yet, it may be interesting to apply a weak field expansion to evaluate the resulting partition function, perhaps to resolve more of the recombination dynamics in the twisted plane.

Finally, brane-brane separation has been suggested as an inflaton candidate for the inflationary scenario in the early universe. Such an action describing lower-dimensional angled Dp-branes can be easily obtained by T-dualizing the above action. In the current scenario of angled branes, toward the end of the inflationary epoch, as the Dp-branes approach each other, the tachyon mode appears. The tachyon rolls as the branes recombine, and the energy released goes to some combination of tachyon matter, closed string modes, open string modes and defects. For the universe to enter the big bang epoch, there are strong limits on the production of tachyon matter, closed string modes and defects. To study the brane dynamics and the conditions imposed by cosmology, an effective action is desirable. It is this application to cosmology that motivates us to construct the effective action for angled branes. Some of these applications will be discussed in a later publication.

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A. Superspace conventions

Since there are many variations on the definitions of the worldsheet superspace quantities, in this appendix we summaries those used in this work. We follow mostly the conventions of [27]. The worldsheet bulk has $\mathcal{N}=(1,1)$ supersymmetry, which can be seen most easily with holomorphic and anti-holomorphic superspace coordinates, $\boldsymbol{z}=(z,\boldsymbol{\theta})$, $\bar{\boldsymbol{z}}=(\bar{z},\bar{\boldsymbol{\theta}})$. Define superderivatives and supercharges

$$\begin{aligned} \mathbf{D}_{\boldsymbol{\theta}} &= \partial_{\boldsymbol{\theta}} + \boldsymbol{\theta}\partial\,, \qquad & \mathbf{D}_{\bar{\boldsymbol{\theta}}} &= \partial_{\bar{\boldsymbol{\theta}}} + \bar{\boldsymbol{\theta}}\bar{\partial}\,, \\ \mathcal{Q}_{\boldsymbol{\theta}} &= \partial_{\boldsymbol{\theta}} - \boldsymbol{\theta}\partial\,, \qquad & \mathcal{Q}_{\bar{\boldsymbol{\theta}}} &= \partial_{\bar{\boldsymbol{\theta}}} - \bar{\boldsymbol{\theta}}\bar{\partial}\,, \end{aligned} \tag{A.1}$$

which satisfy the relations

$$\begin{aligned} &\{\mathbf{D}_{\boldsymbol{\theta}},\mathbf{D}_{\boldsymbol{\theta}}\} = -\{\mathcal{Q}_{\boldsymbol{\theta}},\mathcal{Q}_{\boldsymbol{\theta}}\} = 2\partial\,, \qquad \{\mathbf{D}_{\bar{\boldsymbol{\theta}}},\mathbf{D}_{\bar{\boldsymbol{\theta}}}\} = -\{\mathcal{Q}_{\bar{\boldsymbol{\theta}}},\mathcal{Q}_{\bar{\boldsymbol{\theta}}}\} = 2\bar{\partial}\,, \\ &\{\mathbf{D}_{\boldsymbol{\theta}},\mathbf{D}_{\bar{\boldsymbol{\theta}}}\} = \{\mathcal{Q}_{\boldsymbol{\theta}},\mathcal{Q}_{\bar{\boldsymbol{\theta}}}\} = \{\mathbf{D}_{\boldsymbol{\theta}},\mathcal{Q}_{\boldsymbol{\theta}}\} = \cdots = 0\,. \end{aligned} \tag{A.2}$$

The conformal field theory operator which generates holomorphic supersymmetry transformations is the worldsheet supercurrent,

$$T_F(z) \equiv i\sqrt{\frac{2}{\alpha'}}\psi^\mu \partial X_\mu(z)\,. \tag{A.3}$$

A holomorphic superfield $\mathcal{O}(\boldsymbol{z}) = \mathcal{O}_0(z) + \boldsymbol{\theta}\mathcal{O}_1(z)$ of conformal dimension $(h,0)$ has OPEs with T_F,

$$T_F(z)\mathcal{O}_0(0) = \frac{-\mathcal{O}_1(0)}{z} + \cdots\,, \qquad T_F(z)\mathcal{O}_1(0) = -\frac{2h\mathcal{O}_0(0)}{z^2} - \frac{\partial\mathcal{O}_0}{z} + \cdots\,. \tag{A.4}$$

The anti-holomorphic supercurrent has similar action. The worldsheet bosons and fermions are grouped into a superfield,

$$\boldsymbol{X}^\mu(\boldsymbol{z},\bar{\boldsymbol{z}}) = X^\mu(z,\bar z) + i\boldsymbol{\theta}\sqrt{\frac{\alpha'}{2}}\psi^\mu(z) + i\bar{\boldsymbol{\theta}}\sqrt{\frac{\alpha'}{2}}\tilde\psi^\mu(\bar z) + \boldsymbol{\theta}\bar{\boldsymbol{\theta}}F^\mu(z,\bar z)\,. \tag{A.5}$$

We use the convention that for two complex Grassmanian quantities, $\overline{\boldsymbol{\theta}_1\boldsymbol{\theta}_2} = \bar{\boldsymbol{\theta}}_2\bar{\boldsymbol{\theta}}_1$, so $\boldsymbol{X}$ is a real scalar superfield. It is a simple matter to rewrite the supersymmetric action on the worldsheet,

$$S_\Sigma = \frac{1}{2\pi\alpha'}\int d^2z\; d^2\boldsymbol{\theta}\;\mathbf{D}_{\bar{\boldsymbol{\theta}}}\boldsymbol{X}^\mu\mathbf{D}_{\boldsymbol{\theta}}\boldsymbol{X}_\mu\,,$$

and F can be integrated out being auxiliary.

A.1 Upper half plane

On the disk represented by the upper half plane, along the boundary, $z = \bar z \equiv y$ the 2D $\mathcal{N} = (1,1)$ SUSY reduces to $\mathcal{N} = 1$ SUSY in 1 dimension. Thus the superspace has the boundary $\boldsymbol{\theta} = \pm\bar{\boldsymbol{\theta}}$; choosing the "+" sign, defining $\boldsymbol{\vartheta} = \boldsymbol{\theta}$ and following [43], we see that only the linear combination of supercharges and superderivatives which preserve the boundary conditions are conserved,

$$\mathcal{Q}_{\boldsymbol{\vartheta}} = \mathcal{Q}_{\boldsymbol{\theta}} + \mathcal{Q}_{\bar{\boldsymbol{\theta}}} = \partial_{\boldsymbol{\vartheta}} - \boldsymbol{\vartheta}\partial_y\,, \qquad \mathbf{D}_{\boldsymbol{\vartheta}} = \mathbf{D}_{\boldsymbol{\theta}} + \mathbf{D}_{\bar{\boldsymbol{\theta}}} = \partial_{\boldsymbol{\vartheta}} + \boldsymbol{\vartheta}\partial_y\,,$$

On the boundary because we have $\psi = \pm\tilde\psi$ (where the sign must match that chosen for the supercoordinate), (A.5) reduces to

$$\boldsymbol{X}^\mu(\boldsymbol{y}) = X^\mu(y) + i\boldsymbol{\vartheta}\sqrt{2\alpha'}\psi^\mu(y)\,.$$

The boundary condition on the energy momentum tensor is $T_B(z) = \tilde T_B(\bar z)$. This can be obtained from the methods of [44], by imposing that the conserved worldsheet current crossing the boundary is zero. We can use this boundary condition and that on the coordinates to represent the anti-holomorphic sector as the lower half plane reflection of the holomorphic sector as is usual. Then we obtain a relation between the Virasoro generators,

$$L_m = \oint\limits_\Gamma \frac{dz}{2\pi i}(z-z_0)^{m+1}T_B(z-z_0) = \oint\limits_{\Gamma'} \frac{d\bar z}{2\pi i}(\bar z-\bar z_0)^{m+1}\tilde T_B(\bar z-\bar z_0) = \tilde L_m\,, \tag{A.6}$$

where the contour Γ is closed in the UHP about z_0, and Γ' is its reflection about the real axis. Similar results hold for the the worldsheet supercurrent: the boundary condition that none of the conserved current in a worldsheet SUSY transform flows across the boundary is $T_F(z) = \tilde{T}_F(\bar{z})$, which leads to

$$G_r = \oint_\Gamma \frac{dz}{2\pi i}(z-z_0)^{r+1/2}T_F(z-z_0) = \oint_{\Gamma'} \frac{d\bar{z}}{2\pi i}(\bar{z}-\bar{z}_0)^{r+1/2}\tilde{T}_F(\bar{z}-\bar{z}_0) = \tilde{G}_r\,.$$

Note that these identifications are consistent with the holomorphic and anti-holomorphic super-Virasoro algebras.

From these results, some general statements can be made about operators on the boundary. The condition $L_0 = \tilde{L}_0$ from (A.6) implies that $h = \tilde{h}$, and the condition $G_{-1/2} = \tilde{G}_{-1/2}$ implies that the holomorphic and anti-holomorphic superpartners of a field are equal on the boundary, for instance $\psi = \tilde{\psi}$ and $\partial X = \bar{\partial} X$. That $h = \tilde{h}$ is just a statement that the boundary condition on a $(h,\tilde{h})$ field links it with a $(\tilde{h},h)$ field. Also under the transformation of a field on the boundary to the strip by $z = e^{-iw}$, a $(h,\tilde{h})$ operator on the boundary ($w = i\tau$) acquires a factor

$$\mathcal{O}(\tau) = \left(\frac{\partial w}{\partial z}\right)^{-h}\left(\frac{\partial \bar{w}}{\partial \bar{z}}\right)^{-\tilde{h}}\mathcal{O}(z,\bar{z})|_{z=\bar{z}} = i^{\tilde{h}-h}e^{(h+\tilde{h})\tau}\mathcal{O}(z,\bar{z})|_{z=\bar{z}}\,.$$

Thus, the time propagator along the strip boundary is just the dilatation generator, $L_0+\tilde{L}_0$, on the UHP, as expected.

A.2 Unit disk

It is more common in BSFT to represent the upper half plane by the unit disk centered on the origin. The boundary is defined by $z = 1/\bar{z} \equiv e^{-i\tau}$, and the 2D $\mathcal{N} = (1,1)$ SUSY reduces to $\mathcal{N} = 1$ SUSY in 1 dimension. As discussed in [45] the complex Grassman space has the boundary $\boldsymbol{\theta} = \mp i\bar{\boldsymbol{\theta}}/\bar{z}$. Taking the $-$ sign we define the real Grassman boundary coordinate $\boldsymbol{\vartheta} \equiv (-iz)^{-1/2}\boldsymbol{\theta}$; the linear combination of supercharges and superderivatives which preserve the boundary conditions are conserved,

$$\mathcal{Q}_{\boldsymbol{\vartheta}} = (-iz)^{\frac{1}{2}}\mathcal{Q}_{\boldsymbol{\theta}} + (i\bar{z})^{\frac{1}{2}}\mathcal{Q}_{\bar{\boldsymbol{\theta}}} = \partial_{\boldsymbol{\vartheta}} - \boldsymbol{\vartheta}\partial_\tau\,, \qquad \mathbf{D}_{\boldsymbol{\vartheta}} = (-iz)^{\frac{1}{2}}\mathbf{D}_{\boldsymbol{\theta}} + (i\bar{z})^{\frac{1}{2}}\mathbf{D}_{\bar{\boldsymbol{\theta}}} = \partial_{\boldsymbol{\vartheta}} + \boldsymbol{\vartheta}\partial_\tau\,.$$

This is equivalent to transforming these quantities of conformal dimension 1/2 to the (σ,τ) coordinate system where $z = e^{-\sigma-i\tau}$, and noting that on the boundary ($\sigma = 0$), only the τ components of the supercharge and superderivative are conserved. Note that the branch cut in the definitions of the Grassman quantities in $(\tau,\boldsymbol{\vartheta})$ coordinates ensures that these quantities acquire a "$-$" sign as they go around the disk, as is necessary. The fermion boundary condition is $\psi = i\bar{z}\tilde{\psi}$, and transforming the fermion to the (σ,τ) coordinates of the disk $\psi \to (-iz)^{-1/2}\psi$ with ψ now a real fermion, (A.5) becomes on the boundary

$$\boldsymbol{X}^\mu(\boldsymbol{\tau}) = X^\mu(\tau) + i\boldsymbol{\vartheta}\sqrt{2\alpha'}\psi^\mu(\tau)\,.$$

In the literature on BSFT, there is much variation on these conventions, but these adhere closely to those in [16] (in which α' is set to 2).

The boundary condition on the energy momentum tensor is $z^2 T_B(z) = \bar{z}^2 \tilde{T}_B(\bar{z})$, and that on the supercurrent is $(-i)^1/2 z^{\frac{3}{2}} T_F(z) + i^1/2 \bar{z}^{\frac{3}{2}} \tilde{T}_F(\bar{z}) = 0$. This and the reflection, $z = 1/\bar{z}$, give the relations between the Virasoro and supercurrent generators

$$\begin{aligned} L_m &= \oint_\Gamma \frac{dz}{2\pi i}(z-z_0)^{m+1} T_B(z-z_0) & G_r &= \oint_\Gamma \frac{dz}{2\pi i}(z-z_0)^{r+1/2} T_F(z-z_0) \\ &= -\oint_{\Gamma'} \frac{d\bar{z}}{2\pi i}(\bar{z}-\bar{z}_0)^{-m+1} \tilde{T}_B(\bar{z}-\bar{z}_0) & &= i\oint_{\Gamma'} \frac{d\bar{z}}{2\pi i}(\bar{z}-\bar{z}_0)^{-r+1/2} \tilde{T}_F(\bar{z}-\bar{z}_0) \\ &= -\tilde{L}_{-m}\,, & &= i\tilde{G}_{-r}\,, \end{aligned} \tag{A.7}$$

where the appropriate contours are shown in figure 8. These identifications are again consistent with the holomorphic and anti-holomorphic super-Virasoro algebras, and they imply general statements about boundary operators. The condition $L_0 = -\tilde{L}_0$ from (A.7) implies that $h = -\tilde{h}$, which is the statement that the boundary condition on a $(h, \tilde{h})$ conformal field identifies it with a $(-\tilde{h}, -h)$ conformal field; such an example is ψ with $(h, \tilde{h}) = (1/2, 0)$, and the boundary condition identifies this with $i\bar{z}\tilde{\psi}$ which scales like $(0, -1/2)$. The condition $G_{-1/2} = i\tilde{G}_{1/2}$ links the holomorphic SUSY transformation of an operator with $i\bar{z}$ times its anti-holomorphic SUSY transformation. The fermion boundary condition, $\psi = i\bar{z}\tilde{\psi}$, is again an example. The transformation to polar coordinates parameterizing the disk, $z = e^{-\sigma - i\tau}$, sees a $(h, \tilde{h})$ operator on the boundary $(\sigma = 0)$ acquire the factor

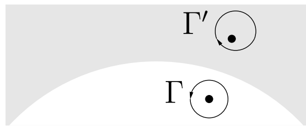

Figure 8: Disk contours.

$$\mathcal{O}(\tau) = \left(\frac{\partial w}{\partial z}\right)^{-h} \left(\frac{\partial \bar{w}}{\partial \bar{z}}\right)^{-\tilde{h}} \mathcal{O}(z,\bar{z})|_{z=1/\bar{z}} = i^{\tilde{h}-h} e^{-i(h-\tilde{h})\tau} \mathcal{O}(z,\bar{z})|_{z=1/\bar{z}}\,.$$

This suggests that the time propagator along the boundary of the disk is just the rotation generator of the plane, $L_0 - \tilde{L}_0$.

On the boundary, both the holomorphic and anti-holomorphic SUSY supercurrents contribute to the SUSY transform of an operator because the boundary condition links the left and right SUSY transformations. The SUSY transformation of a boundary operator is then given by an integral of the OPE of that operator with the supercurrent, where the contour goes half way about the pole location. The result is that for a boundary operator,

$$\begin{aligned} T_F(\tau) &= (-iz)^{3/2} T_F(z) + (i\bar{z})^{3/2} \tilde{T}_F(\bar{z}) = i\sqrt{\frac{2}{\alpha'}}\psi^\mu \dot{X}_\mu(\tau)\,, \\ T_F(\tau)\mathcal{O}_0(0) &= \frac{-\mathcal{O}_1(0)}{\tau} + \cdots\,. \end{aligned} \tag{A.8}$$

A twist superfield can now be constructed using these relations. The boundary field of lowest conformal dimension which keeps T_F single valued while twisting the bosons and fermions is $\mathfrak{T}_0^\pm \equiv (\sigma s)^\pm$. The upper component is given by

$$T_F(\tau)\mathfrak{T}_0^\pm(0) = -\frac{\mathfrak{T}_1^\pm(0)}{\tau} + \cdots\,, \tag{A.9}$$

$$\Longrightarrow \boldsymbol{\mathfrak{T}}^\pm(\boldsymbol{\tau}) = (\sigma s)^\pm(\tau) - \frac{i}{\sqrt{2\alpha'}}\boldsymbol{\vartheta}(\mu u)^\pm(\tau)\,. \tag{A.10}$$

We have used the OPEs (3.1) and (3.2).

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